TECHNICAL REPORT

A. V. ROE CANADA LIMITED

MALTON - ONTARIO

ANALYZED

TECHNICAL DEPARTMENT (Aircraft)

AIRCRAFT: C-105

REPORT NO. 7-0558-8

FILE NO.

NO. OF SHEETS 77.

TITLE: ~~CONFIDENTIAL~~

Classification cancelled / Changed to UNCLASS

By authority of AVRS

Date 30 Sept 96

Signature

Unit / Rank / Appointment AVRSS

DEFLECTION FORMULAS

FORMER — SPAR — STRUT ASSEMBLY.

PREPARED BY V.J. Masi DATE 5-17-55

CHECKED BY DATE

SUPERVISED BY DATE MAY 55.

APPROVED BY DATE

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FORM 1145A

AVRO AIRCRAFT LIMITED

TECHNICAL DEPARTMENT (Aircraft)

AIRCRAFT

REPORT NO. 7-0558-8

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V.J. Masi	5-17-55
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CONFIDENTIAL

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Introduction:

On page 29 the result for δ_1 was first obtained in error; i.e., the coefficient of P_1 was smaller than the coefficient of P_2. In searching for the truth a time came when we felt that the error might be due to the method of reducing the strain energy.

We proved that the method was correct and discovered that the error was arithmetic. We include the proof because it is worth something and ought to appear somewhere; if only to preclude the possibility of its being sought again.

M

FORM 1319A

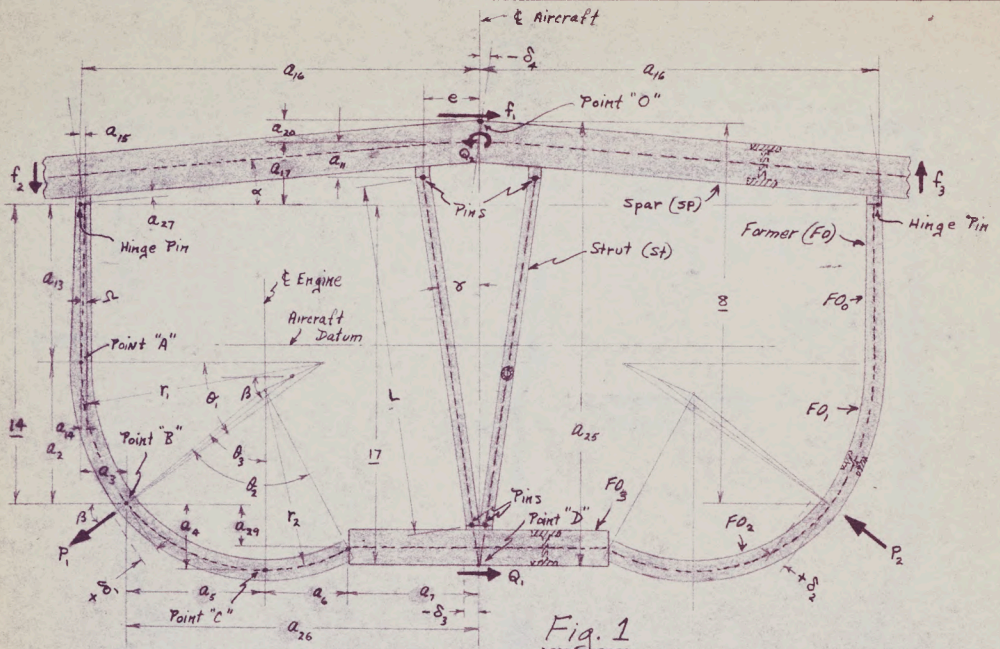

Fig. 1

Note: Forces are positive in the directions shown; P_1 & P_2 are produced by the longerons; Q_1 is produced by shear webs; Q_2 is produced by the fin end/or adjacent structure; f_1, f_2, f_3 are functions of the P's and Q's by statics. Second degree curves are approximated by radii r_1 & r_2 through control points A, B, & C. An underlined number is the name of a constant; e.g. $\underline{7} = C_7 = 19.09189$", etc.

AVRO AIRCRAFT LIMITED
TECHNICAL DEPARTMENT (Aircraft)

AIRCRAFT: C-105

Derivation of Formulas for δ_1; δ_2; δ_3; & a_{29}

REPORT NO. 7-0558-8
SHEET NO. 1

PREPARED BY [illegible] DATE 4-8-55
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AVRO AIRCRAFT LIMITED

TECHNICAL DEPARTMENT (Aircraft)

AIRCRAFT:

REPORT NO. 7-0558-8

SHEET NO. 2

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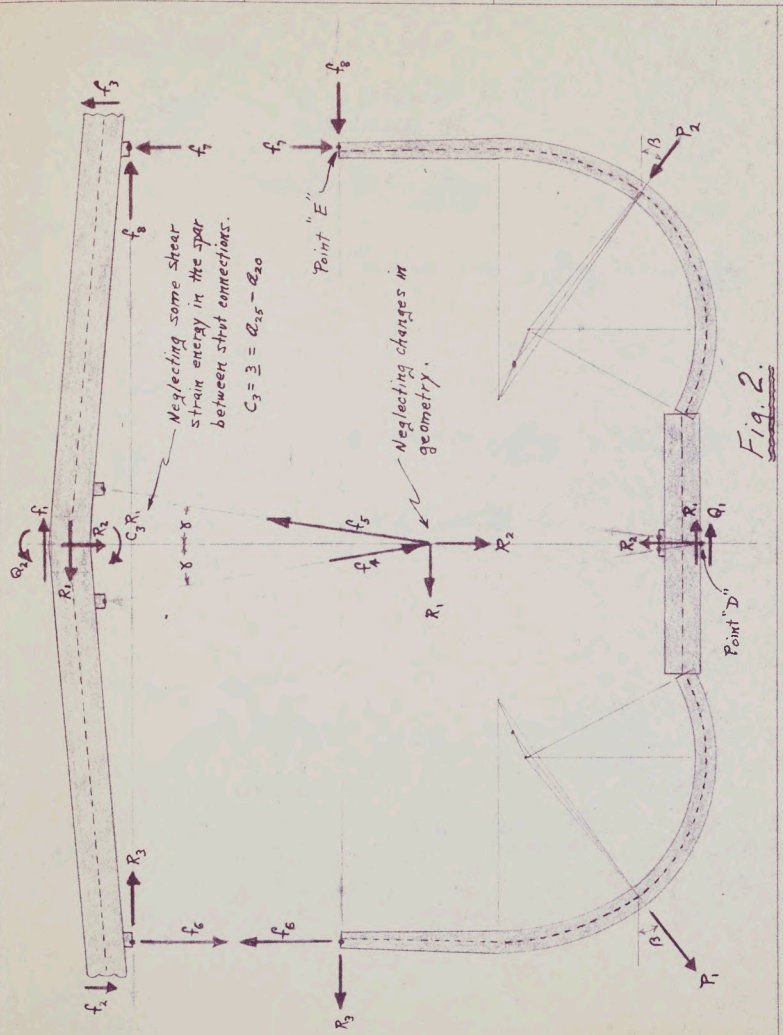
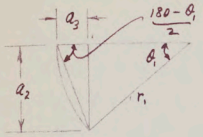


Fig. 2.

FORM 1510A

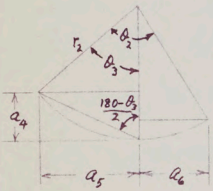
Former Radii and Angles :



$$a_3 = r_1 (1 - \cos\theta_1)$$
$$a_2 = r_1 \sin\theta_1$$

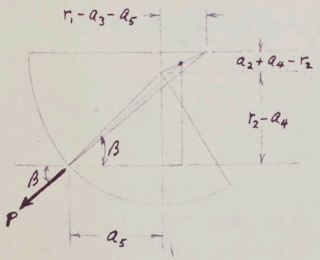
$$\sin\left(90 - \frac{\theta_1}{2}\right) = \frac{a_2}{\sqrt{a_2^2 + a_3^2}}$$

$$\theta_1 = 180 - 2\sin^{-1}\frac{a_2}{\sqrt{a_2^2 + a_3^2}} \quad ; \quad r_1 = \frac{a_2}{\sin\theta_1}$$



$$\theta_3 = 180 - 2\sin^{-1}\frac{a_5}{\sqrt{a_5^2 + a_4^2}} \quad ; \quad r_2 = \frac{a_5}{\sin\theta_3}$$

$$\theta_2 = \theta_3 + \sin^{-1}\frac{a_6}{r_2} .$$



$$\beta = \tan^{-1}\frac{r_2 - a_4 + \frac{a_2 + a_4 - r_2}{2}}{a_5 + \frac{r_1 - a_3 - a_5}{2}}$$

$$= \tan^{-1}\frac{r_2 + a_2 - a_4}{r_1 + a_5 - a_3}$$

Forces f_1---f_8 by statics:

P's & Q's are applied loads; R's are redundants.

Referring to fig. 1,

$F_H = 0 = f_1 + Q_1 - P_1\cos\beta - P_2\cos\beta$.

$F_V = 0 = f_3 - f_2 - P_1\sin\beta + P_2\sin\beta$.

$M_0 = 0 = a_{16}f_2 + Q_2 + a_{16}f_3 + a_{25}Q_1 - \underline{8}P_2\cos\beta - \underline{8}P_1\cos\beta + a_{26}P_2\sin\beta + a_{26}P_1\sin\beta$.

$\underline{6} = \cos\beta \quad ; \quad \underline{7} = \sin\beta$

$f_1 = \underline{6}P_1 + \underline{6}P_2 - Q_1$.

$f_2 - f_3 = -\underline{7}P_1 + \underline{7}P_2$.

$f_2 + f_3 = \frac{\underline{6}\,\underline{8}}{a_{16}}P_1 + \frac{\underline{6}\,\underline{8}}{a_{16}}P_2 - \frac{\underline{7}\,a_{26}}{a_{16}}P_1 - \frac{\underline{7}\,a_{26}}{a_{16}}P_2 - \frac{a_{25}}{a_{16}}Q_1 - \frac{1}{a_{16}}Q_2$.

$f_2 = \frac{1}{2a_{16}}\left[P_1(\underline{6}\,\underline{8} - \underline{7}\,a_{26} - \underline{7}\,a_{16}) + P_2(\underline{6}\,\underline{8} - \underline{7}\,a_{26} + \underline{7}\,a_{16}) - a_{25}Q_1 - Q_2\right]$

$f_3 = \frac{1}{2a_{16}}\left[P_1(\underline{6}\,\underline{8} - \underline{7}\,a_{26} + \underline{7}\,a_{16}) + P_2(\underline{6}\,\underline{8} - \underline{7}\,a_{26} - \underline{7}\,a_{16}) - a_{25}Q_1 - Q_2\right]$

$\underline{8} = a_{20} + a_{17} + a_2 + a_{13} \quad ; \quad \underline{9} = a_3 - a_{14} + a_{15} \quad ; \quad a_{26} = a_{16} - \underline{9}$

$\underline{10} = \frac{\underline{6}\,\underline{8} - (a_{16} - \underline{9})\underline{7} - \underline{7}\,a_{16}}{2a_{16}} = \frac{\underline{6}\,\underline{8} + \underline{7}\,\underline{9}}{2a_{16}} - \underline{7} = \underline{11} - \underline{7}$.

$\underline{11} = \frac{\underline{6}\,\underline{8} - (a_{16} - \underline{9})\underline{7} + \underline{7}\,a_{16}}{2a_{16}} = \frac{\underline{6}\,\underline{8} + \underline{7}\,\underline{9}}{2a_{16}}$.

$\underline{12} = \frac{a_{25}}{2a_{16}} \quad ; \quad \underline{13} = \frac{1}{2a_{16}} \quad ; \quad \underline{14} = a_2 + a_{13} \quad ; \quad \underline{15} = a_{26} + a_{16}$

$$\left.\begin{aligned} f_1 &= \underline{6}P_1 + \underline{6}P_2 - Q_1 \\ f_2 &= \underline{10}P_1 + \underline{11}P_2 - \underline{12}Q_1 - \underline{13}Q_2 \\ f_3 &= \underline{11}P_1 + \underline{10}P_2 - \underline{12}Q_1 - \underline{13}Q_2 \end{aligned}\right\}$$

Referring to fig. 2, $\underline{4} = \frac{1}{2\cos\gamma}$; $\underline{5} = \frac{1}{2\sin\gamma}$; $f_5 - f_4 = \frac{R_2}{\cos\gamma}$, $f_5 + f_4 = \frac{R_1}{\sin\gamma}$

$$\left.\begin{aligned} f_4 &= \underline{5}R_1 - \underline{4}R_2 \\ f_5 &= \underline{5}R_1 + \underline{4}R_2 \end{aligned}\right\}$$

FORM 703A

Referring to fig. 2,

$F_H = 0 = -R_3 - f_8 - \underline{6}P_2 - \underline{6}P_1 + R_1 + Q_1$.

$F_V = 0 = f_6 - f_7 + \underline{7}P_2 - \underline{7}P_1 + R_2$.

$M_E = 0 = -2Q_{16}f_6 - \underline{14}\,\underline{6}P_1 - \underline{14}\,\underline{6}P_2 + \underline{15}\,\underline{7}P_1 - \underline{9}\,\underline{7}P_2 - Q_{16}R_2 + \underline{17}R_1 + \underline{17}Q_1$.

$2Q_{16}f_6 = P_1(-\underline{6}\,\underline{14} + \underline{7}\,\underline{15}) + P_2(-\underline{6}\,\underline{14} - \underline{7}\,\underline{9}) + \underline{17}Q_1 + \underline{17}R_1 - Q_{16}R_2$.

$\underline{18} = \dfrac{-\underline{6}\,\underline{14} + \underline{7}\,\underline{15}}{2Q_{16}}$; $\underline{19} = -\dfrac{\underline{6}\,\underline{14} + \underline{7}\,\underline{9}}{2Q_{16}}$; $\underline{20} = \dfrac{\underline{17}}{2Q_{16}}$.

$$\left.\begin{aligned} f_6 &= \underline{18}P_1 + \underline{19}P_2 + \underline{20}Q_1 + \underline{20}R_1 - .5R_2 \\ f_7 &= \underline{24}P_1 + \underline{23}P_2 + \underline{20}Q_1 + \underline{20}R_1 + .5R_2 \\ f_8 &= -\underline{6}P_1 - \underline{6}P_2 + Q_1 + R_1 - R_3 \end{aligned}\right\}$$

$\underline{24} = \underline{18} - \underline{7}$; $\underline{23} = \underline{19} + \underline{7}$

$$\left.\begin{aligned} f_9 &= f_2 + f_6 = \underline{49}P_1 + \underline{50}P_2 + \underline{51}Q_1 - \underline{13}Q_2 + \underline{20}R_1 - .5R_2 \\ f_{10} &= f_3 + f_7 = \underline{52}P_1 + \underline{53}P_2 + \underline{51}Q_1 - \underline{13}Q_2 + \underline{20}R_1 + .5R_2 \end{aligned}\right\}$$

$\underline{49} = \underline{10} + \underline{18}$
$\underline{50} = \underline{11} + \underline{19}$
$\underline{51} = \underline{20} - \underline{12}$
$\underline{52} = \underline{11} + \underline{24}$
$\underline{53} = \underline{10} + \underline{23}$

Strain Energy Function: "U"

M = Bending moment ; N = Axial force ; V = Shear force.

$$U = \int \frac{M^2}{2EI}\,ds + \int \frac{N^2}{2AE}\,ds + \int \frac{\alpha V^2}{2AG}\,ds \quad ; \quad \alpha = \frac{V_{max.}}{V_{ave.}}$$

Struts:

$M = 0$; $V = 0$; $N = f_4, f_5$

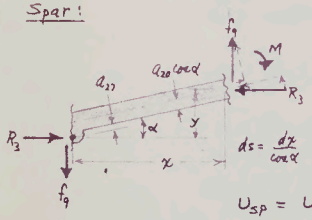
$$U_{ST} = \left[\frac{\underline{25}\,L}{2A_{ST}E_{ST}}\right](f_4^2 + f_5^2)$$

$$= \underline{25}\left[(\underline{5}R_1 - \underline{4}R_2)^2 + (\underline{5}R_1 + \underline{4}R_2)^2\right]$$

$$= \left(2\,\underline{25}^{26}\,\underline{5}^2\right)R_1^2 + \left(2\,\underline{25}^{27}\,\underline{4}^2\right)R_2^2$$

$$= \underline{26}R_1^2 + \underline{27}R_2^2$$

Spar:



$\underline{28} = \tan\alpha$; $\underline{29} = \sin\alpha$; $\underline{30} = \cos\alpha$

$$y = \left(\frac{q_{27} + \underline{30}\,q_{20}}{\underline{30}}\right)^{31} + \underline{28}\,x = \underline{31} + \underline{28}\,x$$

$$U_{SP} = U_{SP_{L.H.}} + U_{SP_{R.H.}}$$

Note: Symetry is such that substitution of f_8 and f_{10} for R_3 and f_9 respectively in $U_{SP_{L.H.}}$ yields $U_{SP_{R.H.}}$

$$M = R_3 y + f_9 x = \underline{31}R_3 + \underline{28}R_3 x + f_9 x = \underline{31}R_3 + (f_9 + \underline{28}R_3)x$$

$$N = \underline{30}R_3 - \underline{29}f_9$$

$$V = \underline{29}R_3 + \underline{30}f_9$$

$$U_{SP_{L.H.}} = \int_0^{q_{16}} \left(\frac{1}{2\,\underline{30}\,E_{SP}I_{SP}}\right)^{32}\left(\underline{31}^2R_3^2 + 2\,\underline{31}R_3(f_9 + \underline{28}R_3)x + (f_9 + \underline{28}R_3)^2x^2\right)dx + \left(\frac{q_{16}}{2\,\underline{30}A_{SP}E_{SP}}\right)^{33}N^2 + \left(\frac{\alpha_{SP}\,q_{16}}{2\,\underline{30}A_{SP}G_{SP}}\right)^{34}V^2$$

$$U_{SP_{L.H.}} = (\underset{35}{32\,31^2}\,a_{16})R_3^2 + (\underset{36}{32\,31}\,a_{16}^2)R_3(f_9 + \underline{28}R_3) + \left[\underset{38}{\frac{33}{3}}\,a_{16}^3\right](f_9 + \underline{28}R_3)^2 + \underline{33}(\underline{10}R_3 - \underline{29}f_9)^2 + \underline{34}(\underline{29}R_3 + \underline{30}f_9)^2 .$$

$$= \underline{35}\,R_3^2 + \underline{36}\,R_3 f_9 + (\underset{37}{\underline{36}\,\underline{28}})R_3^2 + \underline{38}\,f_9^2 + (\underset{299}{2\,\underline{38}\,\underline{28}})R_3 f_9 + (\underset{39}{\underline{38}\,\underline{28}^2})R_3^2 + (\underset{40}{\underline{33}\,\underline{31}^2})R_3^2 - (\underset{-41}{2\,\underline{33}\,\underline{30}\,\underline{29}})R_3 f_9$$
$$+ (\underset{42}{\underline{33}\,\underline{29}^2})f_9^2 + (\underset{43}{\underline{34}\,\underline{29}^2})R_3^2 + (\underset{44}{2\,\underline{34}\,\underline{29}\,\underline{30}})R_3 f_9 + (\underset{45}{\underline{34}\,\underline{30}^2})f_9^2 .$$

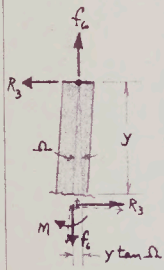
$$\underline{46} = \underline{35} + \underline{37} + \underline{39} + \underline{40} + \underline{43}$$
$$\underline{47} = \underline{36} + \underline{299} - \underline{41} + \underline{44} .$$
$$\underline{48} = \underline{38} + \underline{42} + \underline{45} .$$

$$U_{SP} = \underline{46}\,R_3^2 + \underline{47}\,R_3 f_9 + \underline{48}\,f_9^2 + \underline{46}\,f_8^2 + \underline{47}\,f_8 f_{10} + \underline{48}\,f_{10}^2 .$$

Former:

$$U_{FO} = U_{FO_{L.H.}} + U_{FO_{R.H.}}$$

Note: When $U_{FO_{L.H.}}$ is obtained as a function of R_3; f_6; and P_1; symetry allows $U_{FO_{R.H.}}$ to be written by substitution of f_8; f_7; & P_2; respectively.



Assume the pin is at the centroid of the section and that section properties do not vary along the length a_{13}.

$$\Omega = \tan^{-1}\frac{a_{14} - a_{15}}{a_{13}} = \tan^{-1}\underline{55} \; ; \quad \underline{56} = \sin\Omega \; ; \quad \underline{57} = \cos\Omega$$

$$N = f_6\,\underline{57} - R_3\,\underline{56}$$
$$V = R_3\,\underline{57} + f_6\,\underline{56}$$
$$M = R_3 y + f_6 y\tan\Omega = (R_3 + \underline{55}f_6)\,y \; ; \quad ds = \frac{dy}{\cos\Omega}$$

$$U_{FO_{LH.O}} = \int_0^{a_{13}} \frac{(R_3 + \underline{58}f_6)^2 y^2\,dy}{2E_{FO}I_{FO_0}\,\underline{57}} + \frac{(\underline{57}f_6 - \underline{56}R_3)^2 a_{13}}{2A_{FO_0}E_{FO}\,\underline{57}} + \frac{\alpha_{FO_0}(\underline{57}R_3 + \underline{56}f_6)^2 a_{13}}{2A_{FO_0}G_{FO}\,\underline{57}} .$$

$$= \left(\underset{82}{\frac{a_{13}^3}{6E_{FO}I_{FO_0}\,\underline{57}}}\right)(R_3^2 + 2\,\underline{55}R_3 f_6 + \underline{55}^2 f_6^2) + \left(\underset{83}{\frac{a_{13}}{2A_{FO_0}E_{FO}\,\underline{57}}}\right)(\underline{57}^2 f_6^2 - 2\,\underline{57}\,\underline{56}R_3 f_6 + \underline{56}^2 R_3^2)$$
$$+ \left(\underset{84}{\frac{a_{13}\alpha_{FO_0}}{2A_{FO_0}G_{FO}\,\underline{57}}}\right)(\underline{57}^2 R_3^2 + 2\,\underline{57}\,\underline{56}R_3 f_6 + \underline{56}^2 f_6^2) .$$

$$= R_3^2\left(\underset{85}{\underline{82}} + \underline{83}\,\underline{56}^2 + \underline{84}\,\underline{57}^2\right) + R_3 f_6\left(\underset{58}{2\,\underline{55}\,\underline{82}} - 2\,\underline{56}\,\underline{57}\,\underline{83} + 2\,\underline{56}\,\underline{57}\,\underline{84}\right) + f_6^2\left(\underset{59}{\underline{82}\,\underline{55}^2} + \underline{83}\,\underline{57}^2 + \underline{84}\,\underline{56}^2\right).$$

$$U_{SP_{L.H.}} = \underset{35}{(32\,31^2\,a_{16})}R_3^2 + \underset{36}{(32\,31\,a_{16}^2)}R_3(f_9 + 28R_3) + \underset{38}{\left[\frac{32\,a_{16}^3}{3}\right]}(f_9 + 28R_3)^2 + 33(10R_3 - 29f_9)^2 + 34(29R_3 + 30f_9)^2 .$$

$$= 35\,R_3^2 + 36\,R_3 f_9 + \underset{37}{(36\,28)}R_3^2 + 38\,f_9^2 + \underset{299}{(2\,38\,28)}R_3 f_9 + \underset{39}{(38\,28^2)}R_3^2 + \underset{40}{(33\,30^2)}R_3^2 - \underset{-41}{(2\,33\,30\,29)}R_3 f_9 + \underset{42}{(33\,29^2)}f_9^2 + \underset{43}{(34\,29^2)}R_3^2 + \underset{44}{(2\,34\,29\,30)}R_3 f_9 + \underset{45}{(34\,30^2)}f_9^2 .$$

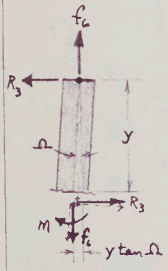
$$46 = 35 + 37 + 39 + 40 + 43 .$$
$$47 = 36 + 299 - 41 + 44 .$$
$$48 = 38 + 42 + 45 .$$

$$U_{SP} = 46\,R_3^2 + 47\,R_3 f_9 + 48\,f_9^2 + 46\,f_8^2 + 47\,f_8 f_{10} + 48\,f_{10}^2 .$$

Former:

$$U_{FO} = U_{FO_{L.H.}} + U_{FO_{R.H.}}$$

Note: When $U_{FO_{L.H.}}$ is obtained as a function of R_3; f_6; and P_1; symetry allows $U_{FO_{R.H.}}$ to be written by substitution of f_8; f_7; & P_2; respectively.



Assume the pin is at the centroid of the section and that section properties do not vary along the length a_{13}.

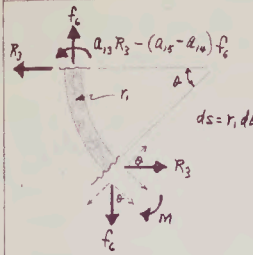
$$\Omega = \tan^{-1}\frac{a_{16} - a_{15}}{a_{13}} = \tan^{-1}\underset{55}{\;} ; \quad \underset{56}{\;} = \sin\Omega \; ; \quad \underset{57}{\;} = \cos\Omega$$

$$N = f_6\,\underset{57}{\;} - R_3\,\underset{56}{\;}$$
$$V = R_3\,\underset{57}{\;} + f_6\,\underset{56}{\;}$$
$$M = R_3 y + f_6 y \tan\Omega = (R_3 + \underset{55}{\;} f_6)y ; \quad ds = \frac{dy}{\cos\Omega}$$

$$U_{FO_{LH.O}} = \int_0^{a_{13}} \frac{(R_3 + \underset{55}{\;} f_6)^2 y^2\,dy}{2E_{FO}I_{FO_0}\,\underset{57}{\;}} + \frac{(\underset{57}{\;} f_6 - \underset{56}{\;} R_3)^2 a_{13}}{2A_{FO_0}E_{FO}\,\underset{57}{\;}} + \frac{\alpha_{FO_0}(\underset{57}{\;} R_3 + \underset{56}{\;} f_6)^2 a_{13}}{2A_{FO_0}G_{FO}\,\underset{57}{\;}} .$$

$$= \left(\frac{a_{13}^3}{6E_{FO}I_{FO_0}\,\underset{57}{\;}}\right)\underset{82}{(}R_3^2 + 2\,\underset{55}{\;}R_3 f_6 + \underset{55^2}{\;}f_6^2\underset{82}{)} + \left(\frac{a_{13}}{2A_{FO_0}E_{FO}\,\underset{57}{\;}}\right)\underset{83}{(}\underset{57^2}{\;}f_6^2 - 2\,\underset{57}{\;}\,\underset{56}{\;}R_3 f_6 + \underset{56^2}{\;}R_3^2\underset{83}{)}$$
$$+ \left(\frac{a_{13}\,\alpha_{FO_0}}{2A_{FO_0}G_{FO}\,\underset{57}{\;}}\right)\underset{84}{(}\underset{57^2}{\;}R_3^2 + 2\,\underset{57}{\;}\,\underset{56}{\;}R_3 f_6 + \underset{56^2}{\;}f_6^2\underset{84}{)} .$$

$$= R_3^2\underset{85}{(}82 + 83\,\underset{56^2}{\;} + 84\,\underset{57^2}{\;}\underset{85}{)} + R_3 f_6\underset{58}{(}2\,\underset{55}{\;}82 - 2\,\underset{56}{\;}\,\underset{57}{\;}83 + 2\,\underset{56}{\;}\,\underset{57}{\;}84\underset{58}{)} + f_6^2\underset{59}{(}82\,\underset{55^2}{\;} + 83\,\underset{57^2}{\;} + 84\,\underset{56^2}{\;}\underset{59}{)} .$$



$$N = R_3 \sin\theta + f_c \cos\theta$$
$$V = R_3 \cos\theta - f_c \sin\theta$$

$$M = a_{13}R_3 - (a_{15} - a_{14})f_c + R_3 r_1 \sin\theta - f_c r_1 (1 - \cos\theta).$$
$$= u + R_3 r_1 \sin\theta + f_c r_1 \cos\theta.$$

$$u = a_{13}R_3 - (r_1 + \overset{86}{a_{15}} - a_{14})f_c$$

$$U_{FO_{LH_1}} = \int_0^{\theta_1} \left\{ \left(\frac{\overset{87}{r_1}}{2E_{FO}I_{FO_1}} \right) \left(u^2 + R_3^2 r_1^2 \sin^2\theta + f_c^2 r_1^2 \cos^2\theta + 2uR_3 r_1 \sin\theta + 2uf_c r_1 \cos\theta + R_3 r_1^2 f_c \sin 2\theta \right) \right.$$
$$\left. + \left(\frac{r_1}{2A_{FO_1}E_{FO}} \right) \left(R_3^2 \sin^2\theta + f_c^2 \cos^2\theta + R_3 f_c \sin 2\theta \right) + \left(\frac{\overset{89}{d_{FO_1}} r_1}{2A_{FO_1}G_{FO}} \right) \left(f_c^2 \sin^2\theta + R_3^2 \cos^2\theta - R_3 f_c \sin 2\theta \right) \right\} d\theta.$$

$$\underline{90} = \int_0^{\theta_1} \sin^2\theta \, d\theta = \frac{\theta}{2} - \frac{\sin 2\theta}{4} \Big]_0^{\theta_1} = \frac{1}{2}\left(\theta_1 - \frac{\sin 2\theta_1}{2}\right).$$

$$\underline{91} = \int_0^{\theta_1} \cos^2\theta \, d\theta = \frac{\theta}{2} + \frac{\sin 2\theta}{4} \Big]_0^{\theta_1} = \frac{1}{2}\left(\theta_1 + \frac{\sin 2\theta_1}{2}\right).$$

$$\underline{92} = \int_0^{\theta_1} \sin\theta \, d\theta = -\cos\theta \Big]_0^{\theta_1} = 1 - \cos\theta_1.$$

$$\underline{93} = \int_0^{\theta_1} \cos\theta \, d\theta = \sin\theta \Big]_0^{\theta_1} = \sin\theta_1.$$

$$\underline{94} = \int_0^{\theta_1} \sin 2\theta \, d\theta = -\frac{1}{2}\cos 2\theta \Big]_0^{\theta_1} = \frac{1}{2}(1 - \cos 2\theta_1).$$

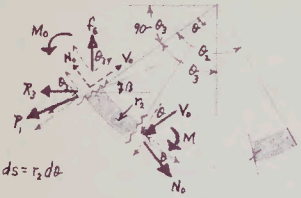
$$U_{FO_{LH_1}} = \underline{87}\left(\theta_1 u^2 + \underline{90} r_1^2 R_3^2 + \underline{91} r_1^2 f_c^2 + 2\underline{92} r_1 R_3 u + 2\underline{93} r_1 f_c u + \underline{94} r_1^2 R_3 f_c\right)$$
$$+ \underline{88}\left(\underline{90} R_3^2 + \underline{91} f_c^2 + \underline{94} R_3 f_c\right) + \underline{89}\left(\underline{90} f_c^2 + \underline{91} R_3^2 - \underline{94} R_3 f_c\right).$$

$$= \left(\underline{87}\,\theta_1 \overset{95}{a_{13}^2}\right) R_3^2 - \left(2\underline{87}\,\theta_1 \overset{-96}{a_{13}}\,\underline{86}\right) R_3 f_c + \left(\underline{87}\,\theta_1 \overset{97}{\underline{86}^2}\right) f_c^2 + \left(\underline{87}\,\overset{98}{\underline{90}}\, r_1^2\right) R_3^2 + \left(\underline{87}\,\overset{99}{\underline{91}}\, r_1^2\right) f_c^2$$
$$+ \left(2\underline{87}\,\overset{100}{\underline{92}}\, r_1 a_{13}\right) R_3^2 - \left(2\underline{87}\,\overset{-101}{\underline{92}}\, r_1 \underline{86}\right) R_3 f_c + \left(2\underline{87}\,\overset{102}{\underline{93}}\, r_1 a_{13}\right) R_3 f_c - \left(2\underline{87}\,\overset{-103}{\underline{93}}\, r_1 \underline{86}\right) f_c^2 + \left(\underline{87}\,\overset{104}{\underline{94}}\, r_1^2\right) R_3 f_c$$
$$+ \left(\underline{88}\,\overset{105}{\underline{90}}\right) R_3^2 + \left(\underline{88}\,\overset{106}{\underline{91}}\right) f_c^2 + \left(\underline{88}\,\overset{107}{\underline{94}}\right) R_3 f_c + \left(\underline{89}\,\overset{108}{\underline{90}}\right) f_c^2 + \left(\underline{89}\,\overset{109}{\underline{91}}\right) R_3^2 - \left(\underline{89}\,\overset{-110}{\underline{94}}\right) R_3 f_c.$$

$$U_{FO_{LH_1}} = R_3^2\left(\underline{95} + \underline{98} + \underline{\underline{111}}{\underline{100}} + \underline{105} + \underline{109}\right) + R_3 f_6\left(-\underline{96} - \underline{101} + \underline{\underline{112}}{\underline{102}} + \underline{104} + \underline{107} - \underline{110}\right) + f_6^2\left(\underline{97} + \underline{99} - \underline{\underline{113}}{\underline{103}} + \underline{106} + \underline{108}\right).$$

$$M_0 = R_3\left(a_{13} + \underline{\underline{14}}{a_2}\right) - f_6\left(a_{15} - \underline{\underline{9}}{a_{14}} + a_3\right)$$

$\underline{6} = \cos\beta$; $\underline{7} = \sin\beta$; $\underline{114} = \cos\theta_3$; $\underline{115} = \sin\theta_3$



$$N_0 = (f_6 - P_1\sin\beta)\sin\theta_3 + (R_3 + P_1\cos\beta)\cos\theta_3 .$$
$$V_0 = (f_6 - P_1\sin\beta)\cos\theta_3 - (R_3 + P_1\cos\beta)\sin\theta_3 .$$

$$N_0 = \underline{115}\left(f_6 - \underline{7}P_1\right) + \underline{114}\left(R_3 + \underline{6}P_1\right) .$$
$$V_0 = \underline{114}\left(f_6 - \underline{7}P_1\right) - \underline{115}\left(R_3 + \underline{6}P_1\right) .$$

$$N_0 = \underline{115}\,u + \underline{114}\,v$$
$$V_0 = \underline{114}\,u - \underline{115}\,v$$

$$N = N_0\cos\theta - V_0\sin\theta$$
$$V = N_0\sin\theta + V_0\cos\theta$$

$$M = M_0 - V_0 r_2\sin\theta - N_0 r_2(1 - \cos\theta) = \left(M_0 - \underline{\underline{w}}{N_0} r_2\right) - V_0 r_2\sin\theta + N_0 r_2\cos\theta .$$

$$U_{FO_{LH_2}} = \int_0^{\theta_2}\left\{\left(\frac{r_2}{2E_{FO_2}I_{FO_2}}\right)\underline{\underline{116}}\left[w^2 + V_0^2 r_2^2\sin^2\theta + N_0^2 r_2^2\cos^2\theta - 2wV_0 r_2\sin\theta + 2wN_0 r_2\cos\theta - V_0 N_0 r_2^2\sin 2\theta\right] + \left(\frac{r_2}{2A_{FO_2}E_{FO_2}}\right)\underline{\underline{117}}\left[V_0^2\sin^2\theta + N_0^2\cos^2\theta - N_0 V_0\sin 2\theta\right] + \left(\frac{r_2\,\alpha_{FO_2}}{2A_{FO_2}G_{FO_2}}\right)\underline{\underline{118}}\left[N_0^2\sin^2\theta + V_0^2\cos^2\theta + N_0 V_0\sin 2\theta\right]\right\} d\theta$$

$$\underline{119} = \int_0^{\theta_2}\sin^2\theta\,d\theta = \frac{1}{2}\left(\theta_2 - \frac{\sin 2\theta_2}{2}\right) .$$

$$\underline{120} = \int_0^{\theta_2}\cos^2\theta\,d\theta = \frac{1}{2}\left(\theta_2 + \frac{\sin 2\theta_2}{2}\right) .$$

$$\underline{121} = \int_0^{\theta_2}\sin\theta\,d\theta = 1 - \cos\theta_2 .$$

$$\underline{122} = \int_0^{\theta_2}\cos\theta\,d\theta = \sin\theta_2 .$$

$$\underline{123} = \int_0^{\theta_2}\sin 2\theta\,d\theta = \frac{1}{2}\left(1 - \cos 2\theta_2\right) .$$

$$U_{FO_{LH_2}} = 116\left(w^2\theta_2 + 119\,V_0^2 r_2^2 + 120\,N_0^2 r_2^2 - 2\,121\,V_0 r_2 w + 2\,122\,N_0 r_2 w - 123\,V_0 N_0 r_2^2\right)$$
$$+ 117\left(119\,V_0^2 + 120\,N_0^2 - 123\,N_0 V_0\right) + 118\left(119\,N_0^2 + 120\,V_0^2 + 123\,N_0 V_0\right).$$

$$= (116\theta_2)^{124} M_0^2 - (2\,116\,\theta_2 r_2)^{125} M_0 N_0 + (116\,\theta_2 r_2^2)^{126} N_0^2 + (116\,119\,r_2^2)^{127} V_0^2 + (116\,120\,r_2^2)^{128} N_0^2 - (2\,116\,121\,r_2^2)^{129} M_0 V_0$$
$$+ (2\,116\,121\,r_2^2)^{130} N_0 V_0 + (2\,116\,122\,r_2)^{131} M_0 N_0 - (2\,116\,122\,r_2^2)^{132} N_0^2 - (116\,123\,r_2^2)^{133} N_0 V_0 + (117\,119)^{134} V_0^2$$
$$+ (117\,120)^{135} N_0^2 - (117\,123)^{136} N_0 V_0 + (118\,119)^{137} N_0^2 + (118\,120)^{138} V_0^2 + (118\,123)^{139} N_0 V_0.$$

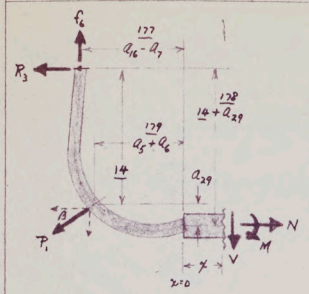
$$= M_0^2(124) + N_0^2(126 + 128^{140} - 132 + 135 + 137) + V_0^2(127 + 134^{141} + 138) + M_0 N_0(-125^{142} + 131)$$
$$+ M_0 V_0(-129) + N_0 V_0(130 - 133^{143} - 136 + 139).$$

$$= 124\left(14^2 R_3^2 + 9^2 f_6^2 - 2\,9\,14\,R_3 f_6\right) + 140\left(115^2 u^2 + 114^2 w^2 + 2\,114\,115\,uw\right)$$
$$+ 141\left(114^2 u^2 + 115^2 w^2 - 2\,114\,115\,uw\right) + 142\left(14\,115\,R_3 u + 14\,114\,R_3 w - 9\,115\,f_6 u - 9\,114\,f_6 w\right)$$
$$- 129\left(14\,114\,R_3 u - 14\,115\,R_3 w - 9\,114\,f_6 u + 9\,115\,f_6 w\right) + 143\left(114\,115\,u^2 + \left[114^2 - 115^2\right]uw - 114\,115\,w^2\right).$$

$$= (124\,14^2)^{144} R_3^2 + (124\,9^2)^{145} f_6^2 - (2\,124\,9\,14)^{146} R_3 f_6 + \left(f_6^2 - 2\,7\,f_6 P_1 + 7^2 P_1^2\right)^{u^2}\Big(140\,115^2 + 141\,114^2 + 143\,114\,115\Big)^{147,148,149}$$
$$+ \left(R_3^2 + 2\,6\,R_3 P_1 + 6^2 P_1^2\right)^{w^2}\Big(140\,114^2 + 141\,115^2 - 143\,114\,115\Big)^{151,152,153,154}$$
$$+ \left(R_3 f_6 + 6 f_6 P_1 - 7 R_3 P_1 - 6\,7 P_1^2\right)^{uw}\Big(2\,140\,114\,115 - 2\,141\,114\,115 + 143\left[114^2 - 115^2\right]\Big)^{155,156,157,158}$$
$$+ \left(R_3 f_6 - 7 R_3 P_1\right)^{R_3 u}\Big(142\,14\,115 - 129\,14\,114\Big)^{159,160,161} + \left(R_3^2 + 6 R_3 P_1\right)^{R_3 w}\Big(142\,14\,114 + 129\,14\,115\Big)^{162,163,164}$$
$$+ \left(f_6^2 - 7 f_6 P_1\right)^{f_6 u}\Big(-142\,9\,115 + 129\,9\,114\Big)^{165,166,167} + \left(R_3 f_6 + 6 f_6 P_1\right)^{f_6 w}\Big(-142\,9\,114 - 129\,9\,115\Big)^{168,169,170}.$$

$$= R_3^2(144^{171} + 154 + 164) + f_6^2(145^{172} + 150 + 167) + P_1^2(7^2\,150^{173} + 6^2\,154 - 6\,7\,158)$$
$$+ R_3 f_6(-146^{174} + 158 + 161 + 170) + R_3 P_1(2\,6\,154 - 7\,158^{175} - 7\,161 + 6\,164)$$
$$+ f_6 P_1(-2\,7\,150^{176} + 6\,158 - 7\,167 + 6\,170).$$

FORM 1109A



$$N = R_3 + P_1 \cos\beta = R_3 + \underline{6}\,P_1$$
$$V = f_6 - P_1 \sin\beta = f_6 - \underline{7}\,P_1$$

$$M = \underline{178}\,R_3 + a_{29}\,\underline{6}\,P_1 - f_6(\underline{177} + x) + \underline{7}\,P_1(\underline{179} + x)\,.$$

$$= \left[\underline{178}\,R_3 + (a_{29}\,\underline{6} + \underline{179}\,\underline{7})P_1 - \underline{177}\,f_6\right] - (f_6 - \underline{7}\,P_1)\,x\,.$$

$$= (\underline{178}\,R_3 + \underset{U}{\underline{180}}\,P_1 - \underline{177}\,f_6) - (f_6 - \underline{7}\,P_1)\,x\,.$$

$$U_{FO_{LH_3}} = \int_0^{a_7} \left(\frac{1}{2E_{FO}I_{FO_3}}\right)\left[u^2 - 2uVx + V^2x^2\right]dx + \left(\frac{a_7}{2A_{FO_3}E_{FO}}\right)N^2 + \left(\frac{a_7 a_{FO_3}}{2A_{FO_3}G_{FO}}\right)V^2$$

$$= \left(\underline{181}\,a_7\right)u^2 - \left(\underline{181}\,\frac{a_7^2}{2}\right)uV + \left(\underline{181}\,\frac{a_7^3}{3} + \underline{183}\right)V^2 + \underline{182}\,N^2.$$

$$= \underline{184}\left(\underline{178}^2R_3^2 + \underline{180}^2P_1^2 + \underline{177}^2f_6^2 + 2\,\underline{178}\,\underline{180}\,R_3P_1 - 2\,\underline{178}\,\underline{177}\,R_3f_6 - 2\,\underline{180}\,\underline{177}\,f_6P_1\right)$$
$$-\underline{185}\left(\underline{178}\,R_3f_6 + \underline{180}\,f_6P_1 - \underline{177}\,f_6^2 - \underline{7}\,\underline{178}\,R_3P_1 - \underline{7}\,\underline{180}\,P_1^2 + \underline{7}\,\underline{177}\,f_6P_1\right)$$
$$+\underline{186}\left(f_6^2 - 2\,\underline{7}\,f_6P_1 + \underline{7}^2P_1^2\right) + \underline{182}\left(R_3^2 + 2\,\underline{6}\,R_3P_1 + \underline{6}^2P_1^2\right).$$

$$= \underset{187}{\left(\underline{184}\,\underline{178}^2\right)}R_3^2 + \underset{188}{\left(\underline{184}\,\underline{180}^2\right)}P_1^2 + \underset{189}{\left(\underline{184}\,\underline{177}^2\right)}f_6^2 + \underset{190}{\left(2\,\underline{184}\,\underline{178}\,\underline{180}\right)}R_3P_1 - \underset{-191}{\left(2\,\underline{184}\,\underline{178}\,\underline{177}\right)}R_3f_6$$
$$-\underset{-192}{\left(2\,\underline{184}\,\underline{180}\,\underline{177}\right)}f_6P_1 - \underset{-193}{\left(\underline{185}\,\underline{178}\right)}R_3f_6 - \underset{-194}{\left(\underline{185}\,\underline{180}\right)}f_6P_1 + \underset{195}{\left(\underline{185}\,\underline{177}\right)}f_6^2 + \underset{196}{\left(\underline{185}\,\underline{7}\,\underline{178}\right)}R_3P_1$$
$$+\underset{197}{\left(\underline{185}\,\underline{7}\,\underline{180}\right)}P_1^2 - \underset{-198}{\left(\underline{185}\,\underline{7}\,\underline{177}\right)}f_6P_1 + \underline{186}\,f_6^2 - \underset{-199}{\left(2\,\underline{186}\,\underline{7}\right)}f_6P_1 + \underset{200}{\left(\underline{186}\,\underline{7}^2\right)}P_1^2 + \underline{182}\,R_3^2$$
$$+\underset{201}{\left(2\,\underline{182}\,\underline{6}\right)}R_3P_1 + \underset{202}{\left(\underline{182}\,\underline{6}^2\right)}P_1^2\,.$$

$$= R_3^2\underset{203}{\left(\underline{187} + \underline{182}\right)} + P_1^2\underset{204}{\left(\underline{188} + \underline{197} + \underline{200} + \underline{202}\right)} + f_6^2\underset{205}{\left(\underline{189} + \underline{195} + \underline{186}\right)}$$
$$+R_3P_1\underset{206}{\left(\underline{190} + \underline{196} + 201\right)} + R_3f_6\underset{207}{\left(-\underline{191} - \underline{193}\right)} + f_6P_1\underset{208}{\left(-\underline{192} - \underline{194} - \underline{198} - \underline{199}\right)}.$$

$$U_{FO_{LH}} = U_{FO_{LH_0}} + U_{FO_{LH_1}} + U_{FO_{LH_2}} + U_{FO_{LH_3}}$$

$$= R_3^2\,(85 + 111 + \overset{209}{171} + 203) + P_1^2\,(173 + \overset{210}{204}) + f_6^2\,(59 + 113 + \overset{211}{172} + 205)$$
$$+ R_3 P_1\,(175 + \overset{212}{206}) + R_3 f_6\,(112 + 58 + \overset{213}{174} + 207) + f_6 P_1\,(176 + \overset{214}{208}).$$

$$U_{FO} = U_{FO_{LH}} + U_{FO_{RH}}$$

$$= 209\,R_3^2 + 209\,f_8^2 + 210\,P_1^2 + 210\,P_2^2 + 211\,f_6^2 + 211\,f_7^2 + 212\,R_3 P_1 + 212\,f_8 P_2$$
$$+ 213\,R_3 f_6 + 213\,f_7 f_8 + 214\,f_6 P_1 + 214\,f_7 P_2.$$

$$U = U_{ST} + U_{SP} + U_{FO} \qquad 300 = 46 + 209$$

$$= 210\,P_1^2 + 210\,P_2^2 + 26\,R_1^2 + 27\,R_2^2 + 300\,R_3^2 + 211\,f_6^2 + 211\,f_7^2 + 300\,f_8^2 + 48\,f_9^2 + 48\,f_{10}^2$$
$$+ 212\,P_1 R_3 + 214\,P_1 f_6 + 214\,P_2 f_7 + 212\,P_2 f_8 + 213\,R_3 f_6 + 47\,R_3 f_9 + 213\,f_7 f_8 + 47\,f_8 f_{10}.$$

$$\frac{\partial U}{\partial X} = \overset{301}{2\,210\,P_1}\frac{\partial P_1}{\partial X} + \overset{301}{2\,210\,P_2}\frac{\partial P_2}{\partial X} + \overset{302}{2\,26\,R_1}\frac{\partial R_1}{\partial X} + \overset{303}{2\,27\,R_2}\frac{\partial R_2}{\partial X} + \overset{304}{2\,300\,R_3}\frac{\partial R_3}{\partial X} + \overset{305}{2\,211\,f_6}\frac{\partial f_6}{\partial X}$$
$$+ \overset{305}{2\,211\,f_7}\frac{\partial f_7}{\partial X} + \overset{304}{2\,300\,f_8}\frac{\partial f_8}{\partial X} + \overset{306}{2\,48\,f_9}\frac{\partial f_9}{\partial X} + \overset{306}{2\,48\,f_{10}}\frac{\partial f_{10}}{\partial X} + 212\,R_3\frac{\partial P_1}{\partial X} + 212\,P_1\frac{\partial R_3}{\partial X}$$
$$+ 214\,f_6\frac{\partial P_1}{\partial X} + 214\,P_1\frac{\partial f_6}{\partial X} + 214\,f_7\frac{\partial P_2}{\partial X} + 214\,P_2\frac{\partial f_7}{\partial X} + 212\,f_8\frac{\partial P_2}{\partial X} + 212\,P_2\frac{\partial f_8}{\partial X} + 213\,f_6\frac{\partial R_3}{\partial X}$$
$$+ 213\,R_3\frac{\partial f_6}{\partial X} + 47\,f_9\frac{\partial R_3}{\partial X} + 47\,R_3\frac{\partial f_9}{\partial X} + 213\,f_8\frac{\partial f_7}{\partial X} + 213\,f_7\frac{\partial f_8}{\partial X} + 47\,f_{10}\frac{\partial f_8}{\partial X} + 47\,f_8\frac{\partial f_{10}}{\partial X}.$$

We treat the partial derivatives, $\frac{\partial P_i}{\partial X}$, $\frac{\partial Q_i}{\partial X}$, and $\frac{\partial R_i}{\partial X}$ as unity when X is P_i, Q_i, and R_i respectively; but as zero when X has other values. $\frac{\partial f_i}{\partial X}$ are constants; and hence $\frac{\partial}{\partial Y}\left(\frac{\partial f_i}{\partial X}\right) = 0$.

FORM 1704-A

$$\frac{\partial^2 U}{\partial X \partial Y} = \frac{\partial P_1}{\partial Y}\left[\underline{301}\frac{\partial P_1}{\partial X} + \underline{212}\frac{\partial P_3}{\partial X} + \underline{214}\frac{\partial f_6}{\partial X}\right] + \frac{\partial P_2}{\partial Y}\left[\underline{301}\frac{\partial P_2}{\partial X} + \underline{214}\frac{\partial f_7}{\partial X} + \underline{212}\frac{\partial f_8}{\partial X}\right]$$
$$+ \frac{\partial P_1}{\partial Y}\left[\underline{302}\frac{\partial P_1}{\partial X}\right] + \frac{\partial P_2}{\partial Y}\left[\underline{303}\frac{\partial P_2}{\partial X}\right] + \frac{\partial P_3}{\partial Y}\left[\underline{304}\frac{\partial P_3}{\partial X} + \underline{212}\frac{\partial P_1}{\partial X} + \underline{213}\frac{\partial f_6}{\partial X} + \underline{47}\frac{\partial f_9}{\partial X}\right]$$
$$+ \frac{\partial f_6}{\partial Y}\left[\underline{305}\frac{\partial f_6}{\partial X} + \underline{214}\frac{\partial P_1}{\partial X} + \underline{213}\frac{\partial P_3}{\partial X}\right] + \frac{\partial f_7}{\partial Y}\left[\underline{305}\frac{\partial f_7}{\partial X} + \underline{214}\frac{\partial P_2}{\partial X} + \underline{213}\frac{\partial f_8}{\partial X}\right]$$
$$+ \frac{\partial f_8}{\partial Y}\left[\underline{304}\frac{\partial f_8}{\partial X} + \underline{212}\frac{\partial P_2}{\partial X} + \underline{213}\frac{\partial f_7}{\partial X} + \underline{47}\frac{\partial f_{10}}{\partial X}\right] + \frac{\partial f_9}{\partial Y}\left[\underline{306}\frac{\partial f_9}{\partial X} + \underline{47}\frac{\partial P_3}{\partial X}\right]$$
$$+ \frac{\partial f_{10}}{\partial Y}\left[\underline{306}\frac{\partial f_{10}}{\partial X} + \underline{47}\frac{\partial f_9}{\partial X}\right].$$

Referring to the formulas for the f's,

$\frac{\partial f_6}{\partial P_1} = \underline{18}$	$\frac{\partial f_7}{\partial P_1} = \underline{24}$	$\frac{\partial f_8}{\partial P_1} = -\underline{6}$	$\frac{\partial f_9}{\partial P_1} = \underline{49}$	$\frac{\partial f_{10}}{\partial P_1} = \underline{52}$
$\frac{\partial f_6}{\partial P_2} = \underline{19}$	$\frac{\partial f_7}{\partial P_2} = \underline{23}$	$\frac{\partial f_8}{\partial P_2} = -\underline{6}$	$\frac{\partial f_9}{\partial P_2} = \underline{50}$	$\frac{\partial f_{10}}{\partial P_2} = \underline{53}$
$\frac{\partial f_6}{\partial Q_1} = \underline{20}$	$\frac{\partial f_7}{\partial Q_1} = \underline{20}$	$\frac{\partial f_8}{\partial Q_1} = 1$	$\frac{\partial f_9}{\partial Q_1} = \underline{51}$	$\frac{\partial f_{10}}{\partial Q_1} = \underline{51}$
$\frac{\partial f_6}{\partial Q_2} = 0$	$\frac{\partial f_7}{\partial Q_2} = 0$	$\frac{\partial f_8}{\partial Q_2} = 0$	$\frac{\partial f_9}{\partial Q_2} = -\underline{13}$	$\frac{\partial f_{10}}{\partial Q_2} = -\underline{13}$
$\frac{\partial f_6}{\partial R_1} = \underline{20}$	$\frac{\partial f_7}{\partial R_1} = \underline{20}$	$\frac{\partial f_8}{\partial R_1} = 1$	$\frac{\partial f_9}{\partial R_1} = \underline{20}$	$\frac{\partial f_{10}}{\partial R_1} = \underline{20}$
$\frac{\partial f_6}{\partial R_2} = -\frac{1}{2}$	$\frac{\partial f_7}{\partial R_2} = \frac{1}{2}$	$\frac{\partial f_8}{\partial R_2} = 0$	$\frac{\partial f_9}{\partial R_2} = -\frac{1}{2}$	$\frac{\partial f_{10}}{\partial R_2} = \frac{1}{2}$
$\frac{\partial f_6}{\partial R_3} = 0$	$\frac{\partial f_7}{\partial R_3} = 0$	$\frac{\partial f_8}{\partial R_3} = -1$	$\frac{\partial f_9}{\partial R_3} = 0$	$\frac{\partial f_{10}}{\partial R_3} = 0$.

Introducing the notations,

$$\underline{UXY} = \frac{\partial^2 U}{\partial X \partial Y} = \underline{UYX}$$

$$\underline{XY} = \frac{\partial X}{\partial Y}.$$

FORM 1193A

$$UP_1 = \delta_1 = UP_1^2 P_1 + UP_1P_2 P_2 + UP_1Q_1 Q_1 + UP_1Q_2 Q_2 + UP_1R_1 R_1 + UP_1R_2 R_2 + UP_1R_3 R_3$$
$$UP_2 = \delta_2 = UP_2P_1 P_1 + UP_2^2 P_2 + UP_2Q_1 Q_1 + UP_2Q_2 Q_2 + UP_2R_1 R_1 + UP_2R_2 R_2 + UP_2R_3 R_3$$
$$UQ_1 = \delta_3 = UQ_1P_1 P_1 + UQ_1P_2 P_2 + UQ_1^2 Q_1 + UQ_1Q_2 Q_2 + UQ_1R_1 R_1 + UQ_1R_2 R_2 + UQ_1R_3 R_3$$
$$UQ_2 = \delta_4 = UQ_2P_1 P_1 + UQ_1P_2 P_2 + UQ_2Q_1 Q_1 + UQ_2^2 Q_2 + UQ_2R_1 R_1 + UQ_2R_2 R_2 + UQ_2R_3 R_3$$

$$UR_1 = 0 = UR_1P_1 P_1 + UR_1P_2 P_2 + UR_1Q_1 Q_1 + UR_1Q_2 Q_2 + UR_1^2 R_1 + UR_1R_2 R_2 + UR_1R_3 R_3$$
$$UR_2 = 0 = UR_2P_1 P_1 + UR_2P_2 P_2 + UR_2Q_1 Q_1 + UR_2Q_2 Q_2 + UR_2R_1 R_1 + UR_2^2 R_2 + UR_2R_3 R_3$$
$$UR_3 = 0 = UR_3P_1 P_1 + UR_3P_2 P_2 + UR_3Q_1 Q_1 + UR_3Q_2 Q_2 + UR_3R_1 R_1 + UR_3R_2 R_2 + UR_3^2 R_3$$

$$UP_1X = 301\,P_1X + 212\,P_2X + 214\,f_6X + 18\,305\,f_4X + 18\,214\,P_1X + 18\,213\,P_3X + 24\,305\,f_7X + 24\,214\,P_2X + 24\,213\,f_8X - 6\,304\,f_8X - 6\,212\,P_2X - 6\,213\,f_7X - 6\,47\,f_{10}X + 49\,306\,f_9X + 49\,47\,P_3X + 52\,306\,f_{10}X + 52\,47\,f_8X$$

$$UP_2X = 301\,P_2X + 214\,f_7X + 212\,f_6X + 19\,305\,f_4X + 19\,214\,P_1X + 19\,213\,P_3X + 23\,305\,f_7X + 23\,214\,P_2X + 23\,213\,f_8X - 6\,304\,f_9X - 6\,212\,P_2X - 6\,213\,f_7X - 6\,47\,f_{10}X + 50\,306\,f_9X + 50\,47\,P_3X + 53\,306\,f_{10}X + 53\,47\,f_8X.$$

$$UQ_1X = 20\,305\,f_4X + 20\,214\,P_1X + 20\,213\,P_3X + 20\,305\,f_7X + 20\,214\,P_2X + 20\,213\,f_8X + 304\,f_9X + 212\,P_2X + 213\,f_7X + 47\,f_{10}X + 51\,306\,f_9X + 51\,47\,P_3X + 51\,306\,f_{10}X + 51\,47\,f_8X.$$

$$UQ_2X = -13\,306\,f_7X - 13\,47\,P_3X - 13\,306\,f_{10}X - 13\,47\,f_8X.$$

$$UR_1X = 302\,R_1X + 20\,305\,f_4X + 20\,214\,P_1X + 20\,213\,P_3X + 20\,305\,f_7X + 20\,214\,P_2X + 20\,213\,f_8X + 304\,f_9X + 212\,P_2X + 213\,f_7X + 47\,f_{10}X + 20\,306\,f_9X + 20\,47\,P_3X + 20\,306\,f_{10}X + 20\,47\,f_8X.$$

$$UR_2X = 303\,R_2X - .5\,305\,f_4X - .5\,214\,P_1X - .5\,213\,P_3X + .5\,305\,f_7X + .5\,214\,P_2X + .5\,213\,f_8X - .5\,306\,f_9X - .5\,47\,P_3X + .5\,306\,f_{10}X + .5\,47\,f_8X.$$

$$UR_3X = 304\,R_3X + 212\,P_1X + 213\,f_6X + 47\,f_7X - 304\,f_8X - 212\,P_2X - 213\,f_7X - 47\,f_{10}X.$$

$$UP_1^2 = 301 + 18\,214 + 18^2\,305 + 18\,214 + 24^2\,305 - 6\,24\,213 + 6^2\,304 - 6\,24\,213 - 6\,47\,52 + 49^2\,306 + 52^2\,306 - 6\,47\,52$$
$$= 301 + 2\,18\,214 + 305\,(18^2+24^2) - 2\,6\,24\,213 + 6^2\,304 - 2\,6\,47\,52 + 306\,(49^2+52^2).$$

$$UP_1P_2 = 19\,214 + 18\,19\,305 + 23\,24\,305 + 24\,214 - 6\,24\,213 + 6^2\,304 - 6\,212 - 6\,23\,213 - 6\,47\,52 + 49\,50\,306 + 52\,53\,306 - 6\,47\,52.$$
$$= 214\,(19+24) + 305\,(18\,19+23\,24) - 6\,213\,(23+24) + 6^2\,304 - 6\,212 - 6\,47\,(52+53) + 306\,(49\,50+52\,53).$$

$UP_1Q_1 = 20\,214 + 18\,20\,305 + 20\,24\,306 + 24\,213 - 6\,304 - 6\,20\,213 - 6\,47\,51 + 49\,51\,306 + 51\,52\,306 + 47\,52.$
$= 20\,214 + 20\,305(18+24) + 213(24-6\,20) - 6\,304 - 47(6\,51-52) + 306\,51(49+52).$

$UP_1Q_2 = 6\,13\,47 - 13\,49\,306 - 13\,52\,306$
$= 6\,13\,47 - 13\,306(49+52)$

$UP_1R_1 = 20\,214 + 18\,20\,305 + 20\,24\,305 + 24\,213 - 6\,304 - 6\,20\,213 - 6\,20\,47 + 20\,49\,306 + 20\,52\,306 + 47\,52.$
$= 20\,214 + 20\,305(18+24) + 213(24-6\,20) - 6\,304 - 47(6\,20-52) + 20\,306(49+52).$

$UP_1R_2 = -.5\,214 - .5\,18\,305 + .5\,24\,305 - .5\,6\,213 - .5\,6\,47 - .5\,49\,306 + .5\,52\,306$
$= -.5\,214 - .5\,305(18-24) - .5\,6\,213 - .5\,6\,47 - .5\,306(49-52).$

$UP_1R_3 = 212 + 18\,213 - 24\,213 + 6\,304 + 47\,49 - 47\,52$
$= 212 + 213(18-24) + 6\,304 + 47(49-52).$

$UP_1^2 = 301 + 23\,214 - 6\,212 + 19^2\,305 + 23^2\,305 + 23\,214 - 6\,23\,213 + 6^2\,304 - 6\,212 - 6\,23\,213$
$- 6\,47\,53 + 50^2\,306 + 53^2\,306 - 6\,47\,53.$
$= 301 + 2\,23\,214 - 2\,6\,212 + 305(19^2+23^2) - 2\,6\,23\,213 + 6^2\,304 - 2\,6\,47\,53 + 306(50^2+53^2).$

$UP_2Q_1 = 20\,214 + 212 + 19\,20\,305 + 20\,23\,305 + 23\,213 - 6\,304 - 6\,20\,213 - 6\,47\,51 + 50\,51\,306$
$+ 51\,53\,306 + 47\,53.$
$= 20\,214 + 212 + 20\,305(19+23) + 213(23-6\,20) - 6\,304 - 47(6\,51-53) + 51\,306(50+53).$

$UP_2Q_2 = 6\,13\,47 - 13\,50\,306 - 13\,53\,306$
$= 6\,13\,47 - 13\,306(50+53).$

$UP_2R_1 = 20\,214 + 212 + 19\,20\,305 + 20\,23\,305 + 23\,213 - 6\,304 - 6\,20\,213 - 6\,20\,47 + 20\,50\,306$
$+ 20\,53\,306 + 47\,53.$

$UP_2R_2 = .5\,214 - .5\,19\,305 + .5\,23\,305 - .5\,6\,213 - .5\,6\,47 - .5\,50\,306 + .5\,53\,306.$
$= .5\,214 - .5\,305(19-23) - .5\,6\,213 - .5\,6\,47 - .5\,306(50-53).$

$UP_2R_3 = -212 + 19\,213 + 23\,213 + 6\,304 + 47\,50 - 47\,53.$
$= -212 + 213(19-23) + 6\,304 + 47(50-53).$

$UQ_1^2 = 20^2 305 + 20^2 305 + 20\,213 + 304 + 20\,213 + 47\,51 + 51^2 306 + 51^2 306 + 47\,51.$
$= 2\,20^2 305 + 2\,20\,213 + 304 + 2\,47\,51 + 2\,51^2 306.$

$UQ_1Q_2 = -1347 - 13\,51\,306 - 13\,51\,306 = -1347 - 2\,13\,51\,306.$

$UQ_1R_1 = 2\,20^2 305 + 2\,20\,213 + 304 + 47(20 + 51) + 2\,20\,51\,306.$

$UQ_1R_2 = .5\,213 + .5\,47.$

$UQ_1R_3 = -304$

$UQ_2^2 = 2\,13^2 306.$

$UQ_2R_1 = -2\,13\,20\,306 - 1347.$

$UQ_2R_2 = 0.$

$UQ_2R_3 = 0.$

$UR_1^2 = 302 + 2\,20^2 305 + 2\,20\,213 + 304 + 2\,20\,47 + 2\,20^2 306.$

$UR_1R_2 = .5\,213 + .5\,47.$

$UR_1R_3 = -304$

$UR_2^2 = 303 + .5\,305 + .5\,306.$

$UR_2R_3 = -213 - 47$

$UR_3^2 = 2\,304.$

Sample Calculation

Using the geometry as shown on sheet no. of the check analysis,

$e = 8.00000$; $d = 3.20000$; $a = 27.00000$; $L = 54.5894$

$a_2 = .707107 \times 27 = 19.09189$
$a_3 = .292893 \times 27 = 7.90811$
$a_4 = 7.90811$
$a_5 = 19.09189$
$a_6 = 0$
$a_7 = 27.00000$
$a_{11} = 3.20000$
$a_{13} = 27.00000$
$a_{14} = 0$
$a_{15} = 0$
$a_{16} = 54.00000$
$a_{17} = 3.20000$
$a_{20} = 3.20000$
$a_{25} = 60.40000$
$a_{26} = 46.09189$
$a_{27} = 0$
$a_{29} = 7.90811$

For another geometry these numbers would be measurements. Although a measurement might be 16.85", it must be carried as 16.85000" in order to insure accuracy when we obtain numbers consisting of few digits by subtraction of two numbers consisting of many digits.

$$\theta_1 = 180 - 2\sin^{-1}\frac{a_2}{\sqrt{a_2^2 + a_3^2}} = 45^\circ$$

$$r_1 = \frac{a_2}{\sin\theta_1} = 27.00000$$

$$\theta_3 = 180 - 2\sin^{-1}\frac{a_5}{\sqrt{a_5^2 + a_4^2}} = 45^\circ$$

$$r_2 = \frac{a_5}{\sin\theta_3} = 27.00000$$

$$\theta_2 = \theta_3 + \sin^{-1}\frac{a_6}{r_2}$$

$$\beta = \tan^{-1}\frac{r_2 + a_2 - a_4}{r_1 + a_5 - a_3}$$

$A_{ST} = .20000\ in^2$; $E_{ST} = 2.90000 \times 10^7$ p.s.i.
$A_{SP} = 4.47100$; $E_{SP} = 1.00000 \times 10^7$; $G_{SP} = .40000 \times 10^7$; $I_{SP} = 33.72000\ in^4$; $\alpha_{SP} = 4.14680$.
$A_{FO} = 1.21040$; $E_{FO} = 1.00000 \times 10^7$; $G_{FO} = .40000 \times 10^7$; $I_{FO} = 1.73820$; $\alpha_{FO} = 5.93100$.

$\underline{3} = a_{25} - a_{20} = 60.4 - 3.2 = 57.20000$

$\underline{4} = \frac{1}{2\cos\delta} = \frac{54.5894}{108} = .505457$

$\underline{5} = \frac{1}{2\sin\delta} = \frac{54.5894}{16} = 3.411838$

$\underline{6} = \cos\beta = .707107$

$\underline{7} = \sin\beta = .707107$

$\underline{8} = a_{20} + a_{17} + a_2 + a_{13} = 3.2 + 3.2 + 19.09189 + 27. = 52.49189$

$\underline{9} = a_3 - a_{14} + a_{15} = 7.90811 - 0 + 0 = 7.90811$

$\underline{10} = \underline{11} - \underline{7} = .395456 - .707107 = -.311651$

$\underline{11} = \frac{\underline{68} + \underline{79}}{2a_{16}} = \frac{.707107(52.49189 + 7.90811)}{108} = .395456$

$\underline{12} = \frac{a_{25}}{2a_{16}} = \frac{60.4}{108} = .559259$

$\underline{13} = \frac{1}{2a_{16}} = \frac{1}{108} = .00925925$

$\underline{14} = a_2 + a_{13} = 19.09189 + 27 = 46.09189$

$\underline{15} = 2a_{16} - \underline{9} = 108 - 7.90811 = 100.09189$

$\underline{16} = \underline{9} = 7.90811$

$\underline{17} = a_{25} - a_{20} - a_{17} = 60.4 - 3.2 - 3.2 = 54.00000$

$\underline{18} = \frac{\underline{715} - \underline{614}}{2a_{16}} = \frac{.707107(100.09189 - 46.09189)}{108} = .353553$

$\underline{19} = -\frac{\underline{614} + \underline{716}}{2a_{16}} = -\frac{.707107(46.09189 + 7.90811)}{108} = -.353553$

$\underline{20} = \frac{\underline{17}}{2a_{16}} = \frac{54}{108} = .50000$

$\underline{21} = \underline{11} = .395456$

$\underline{22} = \underline{10} = -.311651$

$\underline{23} = \underline{19} + \underline{7} = -.353553 + .707107 = .353553$

$\underline{24} = \underline{18} - \underline{7} = .353553 - .707107 = -.353553$

$\underline{25} = \frac{L}{2A_{ST}E_{ST}} = \frac{54.5894}{2 \times .2 \times 2.9 \times 10^7} = 47.05982 \times 10^{-7}$ $\quad 10^{-7} = q$

$\underline{26} = 2\,\underline{25}\,\underline{5}^2 = 2 \times 47.05982\,q\, 3.411838^2 = 1095.613\,q$

$\underline{27} = 2\,\underline{25}\,\underline{4}^2 = 2 \times 47.05982\,q\, .505457^2 = 24.04632\,q$

FORM 3104A

AVRO AIRCRAFT LIMITED

TECHNICAL DEPARTMENT (Aircraft)

REPORT No. 7-0558-8

SHEET No. 19

AIRCRAFT		PREPARED BY	DATE
		M.	
		CHECKED BY	DATE

$\underline{28} = \tan\alpha = 0$

$\underline{29} = \sin\alpha = 0$

$\underline{30} = \cos\alpha = 1.00000$

$\underline{31} = \frac{a_{29}}{\underline{30}} + a_{30} = 0 + 3.2 = 3.20000$

$\underline{32} = \frac{1}{2\,\underline{30}\,E_{sp}\,I_{zp}} = \frac{1}{2 \times 1 \times 10^7 \times 33.72} = .014828\ q$

$\underline{33} = \frac{a_{16}}{2\,\underline{30}\,A_{sp}\,E_{sp}} = \frac{54}{2 \times 1 \times 4.471 \times 10^7} = 6.03890\ q$

$\underline{34} = \frac{a_{16}\,\alpha_{sp}}{2\,\underline{30}\,A_{sp}\,\sigma_{sp}} = \frac{54 \times 4.146}{2 \times 1 \times 4.471 \times .4 \times 10^7} = 62.5932\ q$

$\underline{35} = \underline{32}\,\underline{31}^2 a_{16} = .014828\,q\,3.2^2 \times 54 = 8.19929\ q$

$\underline{36} = \underline{32}\,\underline{31}\,a_{16}^2 = .014828\,q\,3.2 \times 54^2 = 138.363\ q$

$\underline{37} = \underline{32}\,\underline{31}\,a_{16}^2\,\underline{28} = 0$

$\underline{38} = \underline{32}\,\frac{a_{16}^3}{3} = .014828 \times 54^2 \times 18 = 778.292\ q$

$\underline{39} = \underline{38}\,\underline{28}^2 = 0$

$\underline{40} = \underline{33}\,\underline{30}^2 = 6.03890\ q$

$\underline{299} = 2\,\underline{38}\,\underline{28} = 0$

$\underline{41} = 2\,\underline{33}\,\underline{30}\,\underline{29} = 0$

$\underline{42} = \underline{33}\,\underline{29}^2 = 0$

$\underline{43} = \underline{34}\,\underline{29}^2 = 0$

$\underline{44} = 2\,\underline{34}\,\underline{29}\,\underline{30} = 0$

$\underline{45} = \underline{34}\,\underline{30}^2 = 62.5932\ q$

$\underline{46} = \underline{35} + \underline{37} + \underline{39} + \underline{40} + \underline{43} = q(8.19929 + 0 + 0 + 6.03890 + 0) = 14.2382\ q$

$\underline{47} = \underline{36} + \underline{299} - \underline{41} + \underline{44} = q(138.363 + 0 - 0 + 0) = 138.363\ q$

$\underline{48} = \underline{38} + \underline{42} + \underline{45} = q(778.292 + 0 + 62.5932) = 840.885\ q$

$\underline{49} = \underline{10} + \underline{18} = -.311651 + .353553 = .041902$

$\underline{50} = \underline{11} + \underline{19} = .395456 - .353553 = .041903$ } $.0419025$

$\underline{51} = \underline{20} - \underline{12} = .500000 - .559259 = -.059259$

$\underline{52} = \underline{21} + \underline{24} = .395456 - .353553 = .041903$

$\underline{53} = \underline{22} + \underline{23} = -.311651 + .353553 = .041902$ } $.0419025$

$\underline{55} = \tan\Omega = \frac{a_{14} - a_{15}}{a_{13}} = 0 \quad ; \quad \Omega = 0$

$\underline{56} = \sin\Omega = 0$

$\underline{57} = \cos\Omega = 1.00000$

FORM 1N/A

$\underline{82} = \frac{a_{13}^3}{6 E_{FO} I_{FO_0} \underline{57}} = \frac{27^3}{6 \times 10^7 \times 1.7382 \times 1} = 1887.3\ q$

$\underline{83} = \frac{a_{13}}{2 A_{FO_0} E_{FO} \underline{57}} = \frac{27}{2 \times 1.2104 \times 10^7 \times 1} = 11.1533\ q$

$\underline{84} = \frac{a_{13} \alpha_{FO_0}}{2 A_{FO_0} G_{FO} \underline{57}} = \frac{27 \times 5.931}{2 \times 1.2104 \times .4 \times 10^7 \times 1} = 165.376\ q$

$\underline{85} = \underline{82} + \underline{83}\,\underline{56}^2 + \underline{84}\,\underline{57}^2 = q(1887.3 + 0 + 165.376) = 2052.68\ q$

$\underline{58} = 2\,\underline{55}\,\underline{82} + 2\,\underline{56}\,\underline{57}(\underline{84} - \underline{83}) = 0$

$\underline{59} = \underline{82}\,\underline{55}^2 + \underline{83}\,\underline{57}^2 + \underline{84}\,\underline{56}^2 = q(0 + 11.1533 \times 1 + 0) = 11.1533\ q$

$\underline{86} = r_1 + a_{15} - a_{14} = 27 + 0 - 0 = 27.00000$

$\underline{87} = \frac{r_1}{2 E_{FO} I_{FO_1}} = \frac{27}{2 \times 10^7 \times 1.7382} = 7.76665\ q$

$\underline{88} = \frac{r_1}{2 A_{FO_1} E_{FO}} = 11.1533\ q$

$\underline{89} = \frac{r_1 \alpha_{FO_1}}{2 A_{FO_1} G_{FO}} = 165.376\ q$

$\underline{90} = .5(\theta_1 - .5 \sin 2\theta_1) = .5(.7854 - .5) = .14270$

$\underline{91} = .5(\theta_1 + .5 \sin 2\theta_1) = .64270$

$\underline{92} = 1 - \cos\theta_1 = 1 - .707107 = .292893$

$\underline{93} = \sin\theta_1 = .707107$

$\underline{94} = .5(1 - \cos 2\theta_1) = .5(1 - 0) = .50000$

$\underline{95} = \underline{87}\,\theta_1\,a_{13}^2 = 7.76665\ q \cdot .7854 \times 27^2 = 4446.85\ q$

$\underline{96} = 2\,\underline{87}\,\theta_1\,a_{13}\,\underline{86} = 2 \times 4446.85 = 8893.70\ q$

$\underline{97} = \underline{87}\,\theta_1\,\underline{86}^2 = 4446.85\ q$

$\underline{98} = \underline{87}\,\underline{90}\,r_1^2 = 7.76665\ q \cdot .1427 \times 27^2 = 807.949\ q$

$\underline{99} = \underline{87}\,\underline{91}\,r_1^2 = 7.76665\ q \cdot .6427 \times 27^2 = 3638.893\ q$

$\underline{100} = 2\,\underline{87}\,\underline{92}\,r_1\,a_{13} = 2 \times 7.76665\ q \cdot .292893 \times 27^2 = 3316.655\ q$

$\underline{101} = 2\,\underline{87}\,\underline{92}\,r_1\,\underline{86} = 3316.655\ q$

$\underline{102} = 2\,\underline{87}\,\underline{93}\,r_1\,a_{13} = 2 \times 7.76665\ q \cdot .707107 \times 27^2 = 8007.121\ q$

$\underline{103} = 2\,\underline{87}\,\underline{93}\,r_1\,\underline{86} = 8007.121\ q$

$\underline{104} = \underline{87}\,\underline{94}\,r_1^2 = 7.76665\ q \cdot .5 \times 27^2 = 2830.944\ q$

$\underline{105} = \underline{88}\,\underline{90} = 11.1533\ q \cdot .1427 = 1.59158\ q$

$\underline{106} = \underline{88}\,\underline{91} = 11.1533\ q \cdot .6427 = 7.16823\ q$

$\underline{107} = \underline{88}\,\underline{94} = 11.1533\ q \cdot .50000 = 5.57665\ q$

108 = 89 90 = 165.376 q .1427 = 23.5892 q
109 = 89 91 = 165.376 q .6427 = 106.2872 q
110 = 89 94 = 165.376 q .5000 = 82.6880 q
111 = 95 + 98 + 100 + 105 + 109 = q(4446.85 + 807.95 + 3316.66 + 1.59 + 106.29) = 8679.34 q
112 = −96 − 101 + 102 + 104 + 107 − 110 = q(−8893.70 − 3316.66 + 8007.12 + 2830.94 + 6.58 − 82.69) = −1449.41 q
113 = 97 + 99 − 103 + 106 + 108 = q(4446.85 + 3638.89 − 8007.12 + 7.17 + 23.60) = 109.39 q
114 = cos B_3 = .707107
115 = sin B_3 = .707107

116 = $\frac{r_2}{2 E_{FO} I_{FO_2}}$ = 7.76665 q

117 = $\frac{r_2}{2 A_{FO_2} E_{FO}}$ = 11.1533 q

118 = $\frac{r_2 d_{FO}}{2 A_{FO_2} G_{FO}}$ = 165.376 q

119 = .5(B_2 − .5 sin $2B_2$) = .1427
120 = .5(B_2 + .5 sin $2B_2$) = .6427
121 = 1 − cos B_2 = .292893
122 = sin B_2 = .707107
123 = .5(1 − cos $2B_2$) = .50000

124 = 116 B_2 = 7.76665 q .7854 = 6.09993 q
125 = 2 116 B_3 r_2 = 2 × 6.09993 × 27 = 329.396 q
126 = 116 B_2 r_2^2 = 4446.85 q
127 = 116 119 r_2^2 = 807.949 q
128 = 116 120 r_2^2 = 3638.893 q
129 = 2 116 121 r_2 = 122.839 q
130 = 2 116 121 r_2^2 = 3316.655 q
131 = 2 116 122 r_2 = 296.560 q
132 = 2 116 122 r_2^2 = 8007.121 q
133 = 116 123 r_2^2 = 2830.944 q
134 = 117 119 = 1.59158 q
135 = 117 120 = 7.16823 q
136 = 117 123 = 5.57665 q
137 = 118 119 = 23.5792 q
138 = 118 120 = 106.2872 q
139 = 118 123 = 82.6880 q

$140 = 126 + 128 - 132 + 135 + 137 = q(4446.85 + 3638.89 - 8007.12 + 7.17 + 23.60) = 109.39\ q$

$141 = 127 + 134 + 138 = q(807.95 + 1.59 + 106.29) = 915.83\ q$

$142 = -125 + 131 = q(-329.40 + 296.56) = -32.84\ q$

$143 = 130 - 133 - 136 + 139 = q(3316.66 - 2830.94 - 5.58 + 82.69) = 562.83\ q$

$144 = 124\ 14^2 = 6.09993\ q\ 46.09189^2 = 12,959.06\ q$

$145 = 124\ 9^2 = 6.09993\ q\ 7.90811^2 = 381.48\ q$

$146 = 2\ 124\ 9\ 14 = 2 \times 6.09993\ q\ 7.90811 \times 46.09189 = 4446.85\ q$

$147 = 140\ 115^2 = 109.39\ q\ .5000 = 54.695\ q$

$148 = 141\ 114^2 = 915.83\ q\ .5000 = 457.915\ q$

$149 = 143\ 114\ 115 = 562.83\ q\ .5000 = 281.415\ q$

$150 = 147 + 148 + 149 = 794.025\ q$

$151 = 140\ 114^2 = 54.695\ q$

$152 = 141\ 115^2 = 457.915\ q$

$153 = 143\ 114\ 115 = 281.415\ q$

$154 = 151 + 152 - 153 = 231.195\ q$

$155 = 2\ 140\ 114\ 115 = 109.39\ q$

$156 = 2\ 141\ 114\ 115 = 915.83\ q$

$157 = 143\ (114^2 - 115^2) = 0$

$158 = 155 - 156 + 157 = -806.44\ q$

$159 = 142\ 14\ 115 = -32.84\ q\ 46.09189 \times .707107 = -1070.318\ q$

$160 = 129\ 14\ 114 = 122.839\ q\ 46.09189 \times .707107 = 4003.56\ q$

$161 = 159 - 160 = -5073.878\ q$

$162 = 142\ 14\ 114 = -1070.318\ q$

$163 = 129\ 14\ 115 = 4003.56\ q$

$164 = 162 + 163 = 2933.242\ q$

$165 = 142\ 9\ 115 = -32.84\ q\ 7.90811 \times .707107 = -183.637\ q$

$166 = 129\ 9\ 114 = 122.839\ q\ 7.90811 \times .707107 = 686.901\ q$

$167 = -165 + 166 = 870.538\ q$

$168 = 142\ 9\ 114 = -183.637\ q$

$169 = 129\ 9\ 115 = 686.901\ q$

$170 = -168 - 169 = -503.264$

$171 = 144 + 154 + 164 = q(12,959.06 + 231.195 + 2933.242) = 16,123.497\ q$

$172 = 145 + 150 + 167 = q(381.48 + 794.025 + 870.538) = 2046.043\ q$

$173 = 7^2\ 150 + 6^2\ 154 - 6\ 7\ 158 = .5\ q(794.025 + 231.195 + 806.44) = 915.83\ q.$

$174 = -146 + 158 + 161 + 170 = q(-4446.84 - 806.44 - 5073.878 - 503.264) = -10,830.422\ q$

$175 = 2\ 6\ 154 - 7(158 + 161) + 6\ 164 = .707107\ q(462.39 + 806.44 + 5073.878 + 2930.242) = 6559.089\ q$

$176 = -7(2\ 150 + 167) + 6(158 + 170) = .707107(-1588.05 - 870.538 - 806.44 - 503.264) = -2664.586\ q$

FORM 1198A

177 = $a_{16} - a_7 = 54 - 27 = 27$

178 = 14 + $Q_{29} = 46.09189 + 7.90811 = 54.$

179 = $a_5 + a_6 = 19.09189 + 0 = 19.09189$

180 = 6 Q_{24} + 7 179 = .707107 (7.90811 + 19.09189) = 19.09189

181 = $\frac{1}{2 E_{FO} I_{FO_3}}$ = .287654 q

182 = $\frac{a_2}{2 A_{FO_3} E_{FO}}$ = 11.1533 q

183 = $\frac{a_3 \alpha_{FO_3}}{2 A_{FO_3} G_{FO}}$ = 165.376 q

184 = 181 a_7 = 7.76665 q

185 = 184 a_7 = 209.700

186 = 185 $\frac{a_7}{3}$ + 183 = 1887.300 + 165.376 = 2052.676 q

187 = 184 178^2 = 7.76665 q 54^2 = 22,647.551 q

188 = 184 180^2 = 2830.94 q

189 = 184 177^2 = 5661.888 q

190 = 2 184 178 180 = 2 × 7.76665 q 54 × 19.09189 = 16,014.24 q

191 = 2 184 178 177 = 2 × 7.76665 q 54 × 27 = 22,647.55 q

192 = 2 184 180 177 = 2 × 7.76665 q 19.09189 × 27 = 8007.12 q

193 = 185 178 = 209.7 q 54 = 11,323.800 q

194 = 185 180 = 209.7 q 19.09189 = 4003.56 q

195 = 185 177 = 209.7 q 27 = 5661.9 q

196 = 185 7 178 = 209.7 q .707107 × 54 = 8007.12 q

197 = 185 7 180 = 209.7 q .707107 × 19.09189 = 2830.95 q

198 = 185 7 177 = 209.7 q .707107 × 27 = 4003.56 q

199 = 2 186 7 = 1.414214 × 2052.676 = 2902.923 q

200 = 186 7^2 = 1026.338 q

201 = 2 182 6 = 15.7732 q

202 = 182 6^2 = 5.5767 q

203 = 187 + 182 = q (22,647.55 + 11.15) = 22,658.70 q

204 = 188 + 197 + 200 + 202 = q (2830.94 + 2830.94 + 1026.34 + 5.58) = 6693.80 q

205 = 189 + 195 + 186 = q (5661.89 + 5661.89 + 2052.68) = 13,376.46 q

206 = 190 + 196 + 201 = q (16,014.24 + 8007.12 + 15.77) = 24,037.13 q

207 = −191 − 193 = q (−22,647.55 − 11,323.80) = −33,971.35 q

208 = −192 − 194 − 198 − 199 = q (−8007.12 − 4003.56 − 4003.56 − 2902.92) = −18,917.16 q

209 = 85 + 111 + 171 + 203 = q (2052.68 + 8679.34 + 16,123.497 + 22,658.70) = 49,514.217 q

210 = 173 + 204 = q (915.83 + 6693.80) = 7609.63 q

211 = 59 + 113 + 172 + 205 = q (11.15 + 109.39 + 2046.043 + 13,376.46) = 15,543.043

212 = 175 + 206 = q (6559.09 + 24,037.13) = 30,596.22 q

213 = 58 + 112 + 174 + 207 = q (−1449.41 − 10,830.422 − 33,971.35) = − 46,251.182

214 = 176 + 208 = − 2664.59 q − 18,917.16 q = − 21,581.75 q

300 = 209 + 46 = 49,528.455 q

301 = 2 210 = 15,219.26 q

302 = 2 26 = 2,191.226 q

303 = 2 27 = 48.09264 q

304 = 2 300 = 99,056.91 q

305 = 2 211 = 31,086.086 q

306 = 2 48 = 1,681.770 q

UP_1^2 = + 301	=	= 15,219.260
+ 2 18 214	= 2 × .353553 (−21,581.75)	= − 15,260.606
+ 305 (18^2 + 24^2)	= 31,086.086 (.125 + .125)	= 7,771.522
− 2 6 24 213	= −2 × .707107 (−.353553)(− 46,251.182)	= −23,125.591
+ 6^2 304	= .5 × 99,056.91	= 49,528.455
− 2 6 47 52	= −1.414214 × 138.363 × .0419025	= − 8.199
+ 306 (49^2 + 52^2)	= 1681.770 × 2 (.0419025)^2	= 5.906
		34,130.747 q

UP_2 = + 214 (19 + 24)	= − 21,581.75 (−2)(.353553)	= 15,260.606
+ 305 (18 19 + 23 24)	= 31,086.086 (−.125 − .125)	= − 7,771.522
− 6 213 (23 + 24)	= −.707107 (−46,251.182)	= 0.0
+ 6^2 304	=	= 49,528.455
− 6 212	= −.707107 × 30,596.22	= − 21,634.801
− 6 47 (52 + 53)	= − 2 6 47 52	= − 8.199
+ 306 (49 50 + 52 53)	= 306 (49^2 + 52^2)	= 5.906
		35,380.445 q

UP_1Q_1 = + 20 214	= .5 (−21,581.75)	= − 10,790.875
+ 20 305 (18 + 24)	= .5 (31,086.086)(0)	= 0.0
+ 213 (24 − 6 20)	= (−46,251.182)(−.707107)	= 32,704.535
− 6 304	= −.707107 × 99,056.91	= −70,043.834
− 47 (6 51 − 52)	= −138.363 [.707107 (−.059259) − .0419025]	= 11.596
+ 306 51 (49 + 52)	= 1681.770 (−.059259)(2)(.0419025)	= − 8.352
		− 48,136.930

Note: We wish to compare $\frac{\partial U_{P_0}}{\partial R_1}$ with $\frac{\partial U}{\partial Q_1}$.

Considering only the strain energy of the Former (Infinitely Stiff Spar and Struts)

Referring to page 12, $\underline{46} = 0$ $\therefore \underline{300} = \underline{209} = 49,514.217$ g

Also $0 = \underline{26}, \underline{27}, \underline{48}, \underline{47}, 302, 303, 306$ $\underline{304} = 99,028.434$ g

$U\underline{P_1^2} = 34,130.747 - 5.906 + 8.199 - 49,528.455 + 49,514.217 = 34,118.802$ g

$U\underline{P_1P_2} = 35,380.445 - 5.906 + 8.199 - 49,528.455 + 49,514.217 = 35,368.500$ g

$U\underline{P_1Q_1} = -48,126.930 + 8.352 - 11.596 + 70,043.834 - 70,023.699 = -48,110.05$ g

$U\underline{P_1Q_2} = 0$

$U\underline{P_1R_1} = -48,102.825 - 70.47 + 43.12 + 70,043.834 - 70,023.699 = -48,110.05$ g

$U\underline{P_1R_2} = 16,103.62 + 48.919 = 16,152.54$ g

$U\underline{P_1R_3} = 67,935.519 - 70,043.834 + 70,023.699 = 67,915.39$ g

$U\underline{P_2^2} = 37,112.326 - 5.906 + 8.199 - 49,528.455 + 49,514.217 = 37,100.38$ g

$U\underline{P_2Q_1} = -50,235.245 + 8.352 - 11.596 + 70,043.834 - 70,023.699 = -50,218.37$ g

$U\underline{P_2Q_2} = 0$

$U\underline{P_2R_1} = -50,211.14 - 70.47 + 43.12 + 70,043.83 - 70,023.70 = -50,218.36$ g

$U\underline{P_2R_2} = 16,503.029 + 48.919 = 16,551.95$ g

$U\underline{P_2R_3} = 72,152.15 - 70,043.83 + 70,023.70 = 72,132.02$ g

$U\underline{Q_1^2} = 68,344.18 - 11.81 + 16.40 - 99,056.91 + 99,028.43 = 68,320.29$ g

$U\underline{Q_1Q_2} = 0$

$U\underline{Q_1R_1} = 68,310.09 + 99.66 - 60.98 - 99,056.91 + 99,028.43 = 68,320.29$ g

$U\underline{Q_1R_2} = -23,056.411 - 69.18 = -23,125.59$ g

UP_1Q_2 = +6 13 47	= .707107(.00925925)(138.363)	= .9059014
− 13 306 (49 + 52)	= −.00925926(1681.770)(.083805)	= −1.3050052
		− .399104 q
UP_1R_1 = 20 214	=	= −10,790.875
+ 20 305 (18 + 24)	=	= 0.0
+ 213 (24 − 6 20)	=	= 32,704.535
− 6 304	=	= −70,043.834
− 47 (6 20 − 52)	= −138.363(.353553 − .0419025)	= − 43.121
+ 20 306 (49 + 52)	= .5(1681.770)(.083805)	= 70.470
		− 48,102.825 q
UP_1R_2 = −.5 214	=	= 10,790.875
−.5 305 (18 − 24)	= −.5(31,086.086).707107	= −10,990.579
−.5 6 213	= −.353553(−46,251.182)	= 16,352.244
−.5 6 47	= −.353553(138.363)	= − 48.919
−.5 306 (49 − 52)	=	= 0.0
		16,103.621 q
UP_1R_3 = +212	=	= 30,596.22
+ 213 (18 − 24)	= −46,251.182(.707107)	= −32,704.535
+ 6 304	=	= 70,043.834
+ 47 (49 − 52)	=	= 0.0
		67,935.519 q
UP_2^2 = +301	=	= 15,219.26
+ 2 23 214	= .707107 214	= −15,260.606
− 2 6 212	= −1.414214 212	= −43,269.603
+ 305 ($19^2 + 23^2$)	=	= 7,771.522
− 2 6 23 213	= −.5 213	= 23,125.591
+ 6^2 304	= .5 304	= 49,528.455
− 2 6 47 53	=	= − 8.199
+ 306 ($50^2 + 53^2$)	=	= 5.906
		37,112.326 q

$UQ_1R_3 = -99,028.43$ ft $\qquad 0 = UQ_2^2$; UQ_2R_1 ; UQ_2R_2 ; UQ_2R_3

$UR_1^2 = 71,519.25 - 840.89 - 138.36 - 99,056.91 + 99,028.43 - 2,191.23 = 68,320.29$ ft

$UR_1R_2 = -23,125.59$ ft

$UR_1R_3 = -99,028.43$ ft

$UR_2^2 = 16,432.02 - 840.89 = 15,591.13$ ft

$UR_2R_3 = 46,251.18$ ft

$UR_3^2 = 198,056.87$ ft

$0 = u_1 + .00683202\,9\,R_1 - .00231255\,9\,R_2 - .00990284\,3\,R_3$
$0 = u_2 - .00231255\,9\,R_1 + .00155911\,3\,R_2 + .00462511\,8\,R_3$
$0 = u_3 - .00990284\,3\,R_1 + .00462511\,8\,R_2 + .01980568\,7\,R_3$

$0 = 144.8740\,u_1 + R_1 - .3350297\,R_2 - 1.434664\,R_3$
$0 = 432.4213\,u_2 - R_1 + .6741937\,R_2 + 2.000000\,R_3$
$0 = -100.9811\,u_3 + R_1 - .4670495\,R_2 - 2.000000\,R_3$

$.2071442\,R_2 = -432.4213\,u_2 + 100.9811\,u_3$

$R_2 = -2087.538\,u_2 + 487.4918\,u_3$

$.565336\,R_3 + .339164\,R_2 = -144.8740\,u_1 - 432.4213\,u_2$

$.565336\,R_3 = -144.8740\,u_1 - 432.4213\,u_2 + 708.0177\,u_2 - 165.3397\,u_3$

$= -144.8740\,u_1 + 275.5964\,u_2 - 165.3397\,u_3$

$R_3 = -256.2617\,u_1 + 487.4912\,u_2 - 292.4626\,u_3$

$R_1 = 100.9811\,u_3 - 974.9836\,u_2 + 227.6828\,u_3 - 512.5234\,u_1 + 974.9824\,u_2 - 584.9252\,u_3$

$= -512.5234\,u_1 - 256.2613\,u_3$

UP_2Q_1 = + 20 214 = = −10,790.875
+ 212 = = 30,596.22
+ 20 305 (19 + 23) = = 0.0
+ 213 (23 − 6 20) = = 0.0
− 6 304 = = −70,043.834
− 47 (6 51 − 53) = = 11.596
+ 51 306 (50 + 53) = = −8.352
−50,235.245 ℓ

UP_2Q_2 = + 6 13 47 = = .9059014
−13 306 (50 + 53) = = −1.3050052
− .399104 ℓ

UP_2R_1 = 20 214 = = −10,790.875
+ 212 = = 30,596.22
+ 20 305 (19 + 23) = = 0.0
+ 213 (23 − 6 20) = = 0.0
− 6 304 = = −70,043.834
− 47 (6 20 − 53) = = −43.121
+ 20 306 (50 + 53) = = 70.470
−50,211.14 ℓ

UP_3R_2 = .5 214 = = −10,790.875
−.5 305 (19 − 23) = .353553 305 = 10,990.579
−.5 6 213 = = 16,352.244
−.5 6 47 = = −48.919
−.5 306 (50 − 53) = = 0.0
16,503.029 ℓ

UP_3R_3 = −212 = = −30,596.22
+213 (19 − 23) = −.707107 213 = 32,704.535
+ 6 304 = = 70,043.834
+ 47 (50 − 53) = = 0.0
72,152.149 ℓ

$$\frac{\partial U_{FO}}{\partial R_2} = \delta_{3V} = u_2 - .002312559\,R_1 + .001559113\,R_2 + .004625118\,R_3$$

$$= .001610362\,P_1 + .001650303\,P_2 - .002305641\,Q_1$$
$$- .001486448\,P_1 - .001486448\,P_2 + .001975085\,Q_1 - .000017722\,Q_2$$
$$- .000079616\,P_1 + .000079624\,P_2$$
$$- .000044582\,P_1 - .000252971\,P_2 + .000337432\,Q_1 + .000017722\,Q_2$$

$$= -.000000284\,P_1 - .000009492\,P_2 - .000060661\,Q_1$$
(Cannot be correct).

$$\frac{\partial U_{FO}}{\partial R_1} = \delta_3 = u_1 + .006832029\,R_1 - .002312559\,R_2 - .009902843\,R_3$$

$$= -.004811005\,P_1 - .005028836\,P_2 + .006832029\,Q_1$$
$$+ .004391437\,P_1 + .004391437\,P_2 - .005835024\,Q_1 + .000052357\,Q_2$$
$$+ .000118091\,P_1 - .000118102\,P_2$$
$$+ .000095435\,P_1 + .000541636\,P_2 - .000722561\,Q_1 - .000037453\,Q_2$$

$$= -.00020602\,P_1 - .00020686\,P_2 + .00027444\,Q_1 + .00001441 2\,Q_2$$

Note: We obtained δ_3 previously from $\frac{\partial U}{\partial Q_1} = \delta_3$, which was

$$\delta_3 = -.00020869\,P_1 - .00020869\,P_2 + .00027750\,Q_1 + .00001495 8\,Q_2$$

The different results are attributed to the approximation we made in writing the spar energy. The deflection δ_{3V} is extremely small, and cannot be computed in the manner shown above.

$UQ_1^2 = 2\,20^2\,305$	$= .5\,305$	$= 15,543.043$
$+2\,20\,213$	$= 213$	$= -46,251.182$
$+304$	$=$	$= 99,056.91$
$+2\,47\,51$	$= 2(138.363)(-.059259)$	$= -16.399$
$+2\,51^2\,306$	$= 2(.00351129)(1681.770)$	$= 11.812$
		$68,344.184$ q

$UQ_1Q_2 = -13\,47$	$= -.00925925 \times 138.363$	$= -1.281138$
$-2\,13\,51\,306$	$= -2(.00925925)(-.059259)(1681.77)$	$= 1.845554$
		$.56441$ q

$UQ_1R_1 = 2\,20^2\,305$	$=$	$= 15,543.043$
$+2\,20\,213$	$=$	$= -46,251.182$
$+304$	$=$	$= 99,056.91$
$+47\,(20+51)$	$= 138.363\,(.5000-.059259)$	$= 60.98$
$+2\,20\,51\,306$	$= -.059259(1681.77)$	$= -99.66$
		$68,310.091$ q

$UQ_1R_2 = .5\,213$	$=$	$= -23,125.591$
$+.5\,47$	$= .5 \times 138.363$	$= 69.18$
		$-23,056.411$ q

$UQ_1R_3 = -304$	$=$	$= -99,056.91$ q

$UQ_2^2 = 2\,13^2\,306$	$= 2 \times .0000857337 \times 1681.77$	$= .28837$ q

$UQ_2R_1 = -2\,13\,20\,306 - 13\,47 = -.00925925 \times 1681.77 - 1.28114 = -16.85307$ q

$UQ_2R_2 = 0$; $UQ_2R_3 = 0$;

$UR_1^2 = 302$	$=$	$= 2,191.226$
$+2\,20^2\,305$	$=$	$= 15,543.043$
$+2\,20\,213$	$=$	$= -46,251.182$
$+304$	$=$	$= 99,056.91$
$+2\,20\,47$	$=$	$= 138.363$
$+2\,20^2\,306$	$= .5 \times 1681.770$	$= 840.885$
		$71,519.245$ q

$\underline{UR_1R_2} = .5\,\underline{213} + .5\,\underline{47} = -23,125.691_{\text{f}} + 69.18_{\text{f}} = -23,056.411_{\text{f}}$

$\underline{UR_1R_3} = -\underline{304} = -99,056.91_{\text{f}}$

$\underline{UR_2^2} = \underline{303} + .5\,\underline{305} + .5\,\underline{306} = 48.093 + 15,543.043 + 840.885 = 16,432.021_{\text{f}}$

$\underline{UR_2R_3} = -\underline{213} - \underline{47} = 46,251.182 - 138.363 = 46,112.819_{\text{f}}$

$\underline{UR_3^2} = 2\,\underline{304} = 198,113.820_{\text{f}}$

Referring to the set of equations at the top of sheet No. , using the last 3 equations, let u_1 represent the first 4 terms in $\underline{UR_1} = 0$; u_2, in $\underline{UR_2} = 0$; u_3, in $\underline{UR_3} = 0$.

$$\left.\begin{aligned} 0 &= u_1 + .007519245\,R_1 - .002305641\,R_2 - .009905691\,R_3 \\ 0 &= u_2 - .002305641\,R_1 + .0016432021\,R_2 + .0046112819\,R_3 \\ 0 &= u_3 - .009905691\,R_1 + .0046112819\,R_2 + .0198113820\,R_3 \end{aligned}\right\}$$

$$\left.\begin{aligned} 0 &= 139.8225\,u_1 + R_1 - .3223805\,R_2 - 1.385038\,R_3 \\ 0 &= 433.7188\,u_2 - R_1 + .7126876\,R_2 + 2.000000\,R_3 \\ 0 &= -100.9520\,u_3 + R_1 - .4655181\,R_2 - 2.000000\,R_3 \end{aligned}\right\}$$

$$\left.\begin{aligned} -139.8225\,u_1 - 433.7188\,u_2 &= .3803071\,R_2 + .614962\,R_3 \\ 100.9520\,u_3 - 433.7188\,u_2 &= .2471695\,R_2 + \text{——} \end{aligned}\right\}$$

$R_2 = 408.4322\,u_3 - 1754.7423\,u_2$

$.614962\,R_3 = -139.8225\,u_1 - 433.7188\,u_2 - 159.4140\,u_3 + 684.8884\,u_2$

$R_3 = -227.3676\,u_1 + 408.4309\,u_2 - 259.2257\,u_3$

$R_1 = 100.9520\,u_3 + .4655181\,(408.4322\,u_3 - 1754.7423\,u_2) + 2R_3$

$= 100.9520\,u_3$
$+ 190.1326\,u_3 - 816.8643\,u_2$
$- 518.4514\,u_3 + 816.8618\,u_2 - 454.7352\,u_1$

$= -454.7352\,u_1 - .0025\,u_2 - 227.3668\,u_3$

$$R_1 = 2.187404\,P_1 + 2.283277\,P_2 - 3.106300\,Q_1 + .000766351\,Q_2$$
$$- .000004\,P_1 - .000004\,P_2 + .000006\,Q_1$$
$$-1.544628\,P_1 - 1.640500\,P_2 + 2.252225\,Q_1$$

$$= .642772\,P_1 + .642773\,P_2 - .854069\,Q_1 + .000766351\,Q_2$$

$$R_2 = 2.774705\,P_1 + 2.946926\,P_2 - 4.045803\,Q_1$$
$$-2.825770\,P_1 - 2.895856\,P_2 + 4.045806\,Q_1$$

$$= -.051065\,P_1 + .051070\,P_2$$

$$R_3 = 1.093702\,P_1 + 1.141639\,P_2 - 1.553150\,Q_1 + .000383176\,Q_2$$
$$+ .657722\,P_1 + .674035\,P_2 - .941695\,Q_1$$
$$-1.761063\,P_1 - 1.870369\,P_2 + 2.567810\,Q_1$$

$$= -.009639\,P_1 - .054695\,P_2 + .072965\,Q_1 + .000383176\,Q_2$$

Substituting for R's, the first 4 equations on sheet No 14 became;

$$\delta_1 = .00341307\,47\,P_1 + .00353804\,45\,P_2 - .00481269\,3\,Q_1 - .00000003\,9910\,Q_2$$
$$- .00309191\,5\,P_1 - .00309192\,0\,P_2 + .00410831\,3\,Q_1 - .00000036\,86365\,Q_2$$
$$- .00008\,2233\,P_1 + .00008\,2241\,P_2$$
$$- .00006\,5483\,P_1 - .00037\,1573\,P_2 + .00049\,5692\,Q_1 + .00000\,0260313\,Q_2$$

$$= .00017344\,P_1 + .00015679\,P_2 - .00020869\,Q_1 - .00000011232\,Q_2$$

$$\delta_2 = .00353804\,45\,P_1 + .00371123\,26\,P_2 - .00502352\,45\,Q_1 - .00000003\,9911\,Q_2$$
$$- .00322743\,1\,P_1 - .00322743\,6\,P_2 + .00428837\,8\,Q_1 - .00000038\,4794\,Q_2$$
$$- .00008\,4273\,P_1 + .00008\,4281\,P_2$$
$$- .00006\,9547\,P_1 - .00039\,4636\,P_2 + .00052\,6446\,Q_1 + .00000\,0276470\,Q_2$$

$$= .00015679\,P_1 + .00017344\,P_2 - .00020869\,Q_1 - .00000011232\,Q_2$$

When $P_2 = P_1 = P$, & $0 = Q_1 = Q_2$; then $\delta_1 = \delta_2 = .00033023\,P$ (3028. #/IN.)

$P_2 = -P_1 = -P$, & $0 = Q_1 = Q_2$; then $\delta_1 = -\delta_2 = .00001665\,P$ (66,700. #/IN.)

FORM 1193A

$$\delta_3 = -.0048126930\,P_1 - .0050235245\,P_2 + .0068344184\,Q_1 + .0000005644\,Q_2$$
$$+ .0042907814\,P_1 + .0043907828\,P_2 - .0058341531\,Q_1 + .0000052349\,Q_2$$
$$+ .0001177738\,P_1 - .0001177491\,P_2$$
$$+ .0000954\,81\,P_1 + .0000541792\,P_2 - .0007227\,69\,Q_1 - .0000037956\,23\,Q_2$$

$$= -.00020869\,P_1 - .00020869\,P_2 + .00027750\,Q_1 + .00000014958\,Q_2$$

$$\delta_4 = -.0000000039910\,P_1 - .0000000039910\,P_2 + .0000000056441\,Q_1 + .0000000028836\,Q_2$$
$$- .0000108324\,P_1 - .0000108325\,P_2 + .0000143934\,Q_1 - .0000000001292\,Q_2$$

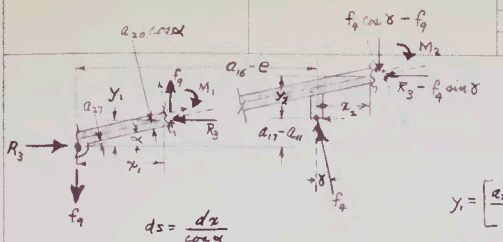
$$= -.00001\,11232\,P_1 - .00001\,11232\,P_2 + .00001\,14958\,Q_1 + .00000027544\,Q_2.$$

Note: If $P_1 = P_2$ & $\delta_3 = 0$; & $Q_2 = 0$,

$$Q_1 = \frac{2 \times .00020869}{.00027750}\,P = 1.50407\,P$$

Then $\delta_1 = .00033023\,P - .00020869 \times 1.5040\,P - .0003\,1388 = .00001635\,P$

This checks with δ_1 when $P_1 = -P_2$ and $Q_1 = 0$, & $Q_2 = 0$.
(Difference is due to strut deflection when $P_1 = -P_2$).



$ds = \frac{dx}{\cos\alpha}$

$\underline{28} = \tan\alpha$
$\underline{29} = \sin\alpha$
$\underline{30} = \cos\alpha$

$$Y_1 = \left[\frac{a_{27} + \underline{30}\,a_{20}}{\underline{30}}\right] + \underline{28}\,x_1 = \underline{31} + \underline{28}\,x_1$$

$$a_{17} - a_{11} + Y_2 = \underline{31} + \underline{28}\left(a_{16} - e + x_2\right)$$

$$Y_2 = \left[\underline{31} + \underline{28}\left(a_{16} - e\right)\right] + \underline{28}\,x_2 - \left(a_{17} - a_{11}\right)$$

Alternate Spar Energy Function:

$N_1 = \underline{30}\,R_3 - \underline{29}\,f_9$
$V_1 = \underline{29}\,R_3 + \underline{30}\,f_9$
$M_1 = R_3 x_1 + f_9 x_1 = \underline{31}\,R_3 + \underline{28}\,R_3 x_1 + f_9 x_1 = \underline{31}\,R_3 + (f_9 + \underline{28}\,R_3)x_1$

$$U_{SP_{LH_1}} = \int_0^{a_{16}-e} \left(\frac{1}{2\underline{30}E_{SP}I_{SP}}\right)M_1^2\,dx_1 + \left(\frac{\overset{62}{60}}{2\underline{30}A_{SP}E_{SP}}\right)N^2 + \left(\frac{\underline{60}\,\alpha_{SP}\overset{63}{G_{SP}}}{2\underline{30}A_{SP}G_{SP}}\right)V^2$$

$$= \int_0^{\overset{60}{a_{16}-e}} \left[\underline{31}^2R_3^2 + 2\underline{31}R_3(f_9 + \underline{28}R_3)x_1 + (f_9 + \underline{28}R_3)^2x_1^2\right]dx + \underline{62}\,N^2 + \underline{63}\,V^2$$

$$= \left(\underline{32}\,\underline{31}^2\,\underline{60}\right)R_3^2 + \left(\underline{32}\,\underline{31}\,\overset{65}{60^2}\right)R_3(f_9 + \underline{28}R_3) + \left(\frac{\underline{32}\,\overset{66}{60^3}}{3}\right)(f_9 + \underline{28}R_3)^2 + \underline{62}\,N^2 + \underline{63}\,V^2$$

$$= \underline{64}\,R_3^2 + \underline{65}\,R_3 f_9 + \left(\underline{65}\,\underline{28}\overset{67}{)}R_3^2 + \underline{66}\,f_9^2 + \left(2\,\underline{28}\,\underline{66}\overset{68}{)}R_3 f_9 + \left(\underline{66}\,\underline{28}^2\overset{69}{)}R_3^2 + \left(\underline{62}\,\underline{30}^2\overset{70}{)}R_3^2 - \left(2\,\underline{29}\,\underline{30}\,\underline{62}\overset{71}{)}R_3 f_9$$
$$+ \left(\underline{62}\,\underline{29}^2\overset{72}{)}f_9^2 + \left(\underline{63}\,\underline{29}^2\overset{73}{)}R_3^2 + \left(2\,\underline{29}\,\underline{30}\,\underline{63}\overset{74}{)}R_3 f_9 + \left(\underline{63}\,\underline{30}^2\overset{75}{)}f_9^2.$$

$$= R_3^2\left(\underline{64} + \underline{67} + \underline{69} + \underline{70} + \overset{76}{\underline{73}}\right) + R_3 f_9\left(\underline{65} + \overset{77}{\underline{68}} - \underline{71} + \underline{74}\right) + f_9^2\left(\underline{66} + \overset{78}{\underline{72}} + \underline{75}\right).$$

$N_2 = (f_4\cos\delta - f_9)\sin\alpha + (R_3 - f_4\sin\delta)\cos\alpha$ $\qquad \underline{307} = \sin\delta\ ;\ \underline{308} = \cos\delta$
$V_2 = (f_4\cos\delta - f_9)\cos\alpha - (R_3 - f_4\sin\delta)\sin\alpha$
$M_2 = R_3\left(\underline{61} + \underline{28}\,x_2\right) + f_9\left(a_{16} - e + x_2\right) - f_4\cos\delta\,x_2 - f_4\sin\delta\left[\underline{61} + \underline{28}\,x_2 - (a_{17} - a_{11})\right].$

$$N_2 = \left(\underline{308}\overset{U}{f_4} - f_9\right)\underline{29} + \left(R_3 - \underline{307}\overset{N}{f_4}\right)\underline{30} = \underline{29}\,U + \underline{30}\,N$$
$$V_2 = \left(\underline{308}f_4 - f_9\right)\underline{30} - \left(R_3 - \underline{307}f_4\right)\underline{29} = \underline{30}\,U - \underline{29}\,N$$

$$M_2 = x_2\left[\underline{28}\,R_3 + \overset{-U}{f_9 - f_4\,\underline{308}} - f_4\,\underline{307}\,\underline{28}\right] + \underline{61}\left(R_3 - \underline{307}\overset{N}{f_4}\right) + (a_{16} - e)(f_9) + (a_{17} - a_{11})\,\underline{307}\,f_4$$
$$= (\underline{28}\,N - U)\,x_2 + \left[\underline{61}\,N + f_9\overset{60}{(a_{16} - e)} + \underline{307}\,f_4\overset{307}{(a_{17} - a_{11})}\right]$$
$$= \underline{\qquad\qquad} N$$

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$$U_{SP_{LH_2}} = \int_0^e 32\left[(28w-u)^2\chi_2^2 + 2w(28w-u)\chi_2 + w^2\right]d\chi_2 + \left[\frac{e}{60}\,62\right]N^2 + \left[\frac{e}{60}\,63\right]V^2$$

$$= \left(\underset{312}{32}\,\frac{e^3}{3}\right)\left(\underset{312}{28^2}w^2 - 2\,\underset{312}{28}\,uw + u^2\right) + \left(\underset{313}{32}\,e^3\,\underset{313}{28}\right)ww - \left(\underset{-314}{32}\,e^2\right)uw + \left(\underset{315}{32}\,e\right)w^2$$
$$+ \underset{316}{310}\,\underset{316}{29^2}u^2 + 2\,\underset{317}{310}\,\underset{317}{29}\,\underset{317}{30}\,uw + \underset{318}{310}\,\underset{318}{30^2}w^2 + \underset{319}{311}\,\underset{319}{30^2}u^2 - 2\,\underset{-320}{311}\,\underset{-320}{29}\,\underset{-320}{30}\,uw + \underset{321}{311}\,\underset{321}{29^2}w^2.$$

$$= \underset{322}{312}\,\underset{322}{28^2}w^2 - 2\,\underset{-323}{312}\,\underset{-323}{28}\,uw + \underset{312}{312}\,u^2 + \underset{313}{313}\,ww - \underset{-314}{314}\,uw + \underset{315}{315}\,w^2 + \underset{316}{316}\,u^2$$
$$+ \underset{317}{317}\,uw + \underset{318}{318}\,w^2 + \underset{319}{319}\,u^2 - \underset{-320}{320}\,uw + \underset{321}{321}\,w^2.$$

$$= w^2\left(\underset{322}{322} + \underset{324}{324} + \underset{321}{321}\right) + u^2\left(\underset{312}{312} + \underset{316}{316} + \underset{319}{319}\right) + w^2\left(\underset{315}{315}\right) + uw\left(-\underset{323}{323} + \underset{317}{317} - \underset{320}{320}\right)$$
$$+ ww\left(\underset{313}{313}\right) + uw\left(-\underset{-314}{314}\right).$$

$$= \underset{324}{324}\,w^2 + \underset{325}{325}\,u^2 + \underset{326}{326}\,uw + \underset{327}{315}\,\underset{327}{61^2}w^2 + \underset{328}{315}\,\underset{328}{60^2}f_g^2 + \underset{329}{315}\,\underset{329}{307^2}\,\underset{329}{309^2}f_k^2$$
$$+ 2\,\underset{330}{315}\,\underset{330}{61}\,\underset{330}{60}f_g w + 2\,\underset{331}{315}\,\underset{331}{61}\,\underset{331}{307}\,\underset{331}{309}f_k w + 2\,\underset{332}{315}\,\underset{332}{60}\,\underset{332}{307}\,\underset{332}{309}f_k f_g + \underset{333}{313}\,\underset{333}{61}w^2$$
$$+ \underset{334}{313}\,\underset{334}{60}f_g w + \underset{335}{313}\,\underset{335}{307}\,\underset{335}{309}f_k w - \underset{-336}{314}\,\underset{-336}{61}\,uw - \underset{-337}{314}\,\underset{-337}{60}f_g u - \underset{-338}{314}\,\underset{-338}{307}\,\underset{-338}{309}f_k u.$$

$$= w^2\left(\underset{339}{324} + \underset{339}{327} + \underset{339}{333}\right) + u^2\left(\underset{325}{325}\right) + uw\left(\underset{340}{326} - \underset{340}{336}\right) + f_g^2\left(\underset{328}{328}\right) + f_k^2\left(\underset{329}{329}\right)$$
$$+ f_k f_g\left(\underset{332}{332}\right) + f_g w\left(\underset{341}{330} + \underset{341}{334}\right) + f_k w\left(\underset{342}{331} + \underset{342}{335}\right) + f_g u\left(-\underset{-337}{337}\right) + f_k u\left(-\underset{-338}{338}\right).$$

$$= \underset{339}{339}\left(R_3^2 - 2\,\underset{339}{307}\,R_3 f_k + \underset{339}{307^2}f_k^2\right) + \underset{325}{325}\left(\underset{325}{308^2}f_k^2 - 2\,\underset{325}{308}\,f_k f_g + f_g^2\right)$$
$$+ \underset{340}{340}\left(\underset{340}{308}\,R_3 f_k - \underset{340}{307}\,\underset{340}{308}\,f_k^2 - R_3 f_g + \underset{340}{307}\,f_k f_g\right) + \underset{328}{328}\,f_g^2 + \underset{329}{329}\,f_k^2 + \underset{332}{332}\,f_k f_g$$
$$+ \underset{341}{341}\,R_3 f_g - \underset{341}{341}\,\underset{341}{307}\,f_k f_g + \underset{342}{342}\,R_3 f_g - \underset{342}{342}\,\underset{342}{307}\,f_k^2 - \underset{337}{337}\,\underset{337}{308}\,f_k f_g + \underset{337}{337}\,f_g^2 - \underset{338}{338}\,\underset{338}{308}\,f_k^2 + \underset{336}{336}\,f_k f_g.$$

$$= R_3^2\left(\underset{339}{339}\right) + R_3 f_k\left(-2\,\underset{-343}{307}\,\underset{339}{339} + \underset{344}{340}\,\underset{344}{308} + \underset{345}{342}\right) + f_k^2\left(\underset{346}{339}\,\underset{346}{307^2} + \underset{347}{325}\,\underset{347}{308^2}\right)$$
$$- \underset{-348}{307}\,\underset{-348}{308}\,\underset{-348}{340} + \underset{349}{329} - \underset{-349}{342}\,\underset{-349}{307} - \underset{-350}{338}\,\underset{-350}{308}\Big) + f_k f_g\left(-2\,\underset{-352}{308}\,\underset{352}{325} + \underset{353}{307}\,\underset{353}{340} + \underset{356}{332}\right.$$
$$\left. - \underset{-354}{341}\,\underset{-354}{307} - \underset{-355}{337}\,\underset{-355}{308} + \underset{358}{338}\right) + f_g^2\left(\underset{357}{325} + \underset{357}{328} + \underset{357}{337}\right) + R_3 f_g\left(-\underset{-358}{340} + \underset{358}{341}\right).$$

$$U_{SP_{LH}} = R_3^2\left(\underset{359}{76} + \underset{359}{339}\right) + R_3 f_g\left(\underset{360}{358} + \underset{360}{77}\right) + f_g^2\left(\underset{361}{78} + \underset{361}{357}\right) + R_3 f_k\left(\underset{345}{345}\right) + f_k^2\left(\underset{351}{351}\right) + f_k f_g\left(356\right)$$

$$U_{SP} = \underset{359}{359}\,R_3^2 + \underset{359}{359}\,f_8^2 + \underset{360}{360}\,R_3 f_9 + \underset{360}{360}\,f_8 f_{10} + \underset{361}{361}\,f_9^2 + \underset{361}{361}\,f_{10}^2 + \underset{345}{345}\,R_3 f_4 + \underset{345}{345}\,f_8 f_5 + \underset{351}{351}\,f_4^2 + \underset{351}{351}\,f_5^2$$
$$+ \underset{356}{356}\,f_4 f_9 + \underset{356}{356}\,f_5 f_{10}$$

Proof of Strain Energy Reduction Technique:

The strain energy for a system with applied loads $P_1 \ldots\ldots P_n$; and internal redundants $R_1 \ldots\ldots R_m$; is of the form

$$U = \underline{11}\,P_1^2 + \underline{12}\,P_1P_2 + \ldots\ldots + \underline{1n}\,P_1P_n \;;\; + \underline{11}\,P_1R_1 + \underline{12}\,P_1R_2 + \ldots\ldots + \underline{1m}\,P_1R_m$$
$$+ \underline{22}\,P_2^2 + \ldots\ldots + \underline{2n}\,P_2P_n \;;\; + \underline{21}\,P_2R_1 + \underline{22}\,P_2R_2 + \ldots\ldots + \underline{2m}\,P_2R_m$$
$$\ddots \qquad\qquad \vdots \qquad\qquad \vdots$$
$$+ \underline{nn}\,P_n^2 \;;\; + \underline{n1}\,P_nR_1 + \underline{n2}\,P_nR_2 + \ldots\ldots + \underline{nm}\,P_nR_m \;;$$
$$+ \underline{11}\,R_1^2 + \underline{12}\,R_1R_2 + \ldots\ldots + \underline{1m}\,R_1R_m$$
$$+ \underline{22}\,R_2^2 + \ldots\ldots + \underline{2m}\,R_2R_m$$
$$\ddots \qquad\qquad \vdots$$
$$+ \underline{mm}\,R_m^2 \,.$$

where underlined numbers are constants, some of which may be zero.

If we consider only those terms which contain P_i when we take $\left[\frac{\partial U}{\partial P_i}\right]_o$; i.e., we treat $\frac{\partial R_j}{\partial P_i}$ as zero even though we know that

$R_j = \frac{\partial R_j}{\partial P_1}P_1 + \frac{\partial R_j}{\partial P_2}P_2 + \ldots\ldots + \frac{\partial R_j}{\partial P_n}P_n$, then the displacement

$$\delta_i = \left[\frac{\partial U}{\partial P_i}\right]_o = \underline{i1}\,P_1 + \underline{2i}\,P_2 + \ldots + 2\underline{ii}\,P_i + \ldots + \underline{in}\,P_n \;;\; + \underline{i1}\,R_1 + \underline{i2}\,R_2 + \ldots\ldots + \underline{im}\,R_m \,.$$

$$0 = \left[\frac{\partial U}{\partial R_i}\right]_o = \underline{i1}\,P_1 + \underline{2i}\,P_2 + \ldots + \underline{ni}\,P_n \;;\; + \underline{i1}\,R_1 + \underline{2i}\,R_2 + \ldots + 2\underline{ii}\,R_i + \ldots\ldots + \underline{im}\,R_m \,.$$

Thus we obtain $(n+m)$ equations involving n P's and m R's. The last m equations can be transformed into the n equations

$$R_j = \left[\frac{\partial R_j}{\partial P_1}\right]_o P_1 + \left[\frac{\partial R_j}{\partial P_2}\right]_o P_2 + \ldots\ldots + \left[\frac{\partial R_j}{\partial P_n}\right]_o P_n \,.$$

Now substituting for the R's in the first n equations,

$$\delta_{i_o} = P_1\left\{\underline{i1} + \underline{i1}\left[\frac{\partial R_1}{\partial P_1}\right]_o + \ldots + \underline{im}\left[\frac{\partial R_m}{\partial P_1}\right]_o\right\} + \ldots + P_n\left\{\underline{in} + \underline{i1}\left[\frac{\partial R_1}{\partial P_n}\right]_o + \ldots\ldots + \underline{im}\left[\frac{\partial R_m}{\partial P_n}\right]_o\right\} \,.$$

We shall now differentiate U with respect to P_i recognizing that every R is a function of P_i . After subtraction of the terms shown above in δ_{i_o} , the remainder will be shown to be zero.

$$\frac{\partial U}{\partial P_i} = \underline{11}\,P_1 + \underline{21}\,P_2 + \cdots + 2\underline{i1}\,P_i + \cdots + \underline{1m}\,P_m \;;\; + \underline{i1}\,R_1 + \underline{i2}\,R_2 + \cdots + \underline{im}\,R_m \;,\; \leftarrow \text{This} = \delta_{i_0}$$

$$+ \frac{\partial R_1}{\partial P_i}\left(\underline{11}\,P_1 + \underline{21}\,P_2 + \cdots + \underline{m1}\,P_m \;;\; + 2\underline{11}\,R_1 + \underline{12}\,R_2 + \cdots + \underline{1m}\,R_m\right) ; \leftarrow \text{This} = \frac{\partial R_1}{\partial P_i}\left[\frac{\partial U}{\partial R_1}\right]_0$$

$$+ \frac{\partial R_2}{\partial P_i}\left(\underline{12}\,P_1 + \underline{22}\,P_2 + \cdots + \underline{m2}\,P_m \;;\; + \underline{12}\,R_1 + 2\underline{22}\,R_2 + \cdots + 2\underline{2m}\,R_m\right) ; \leftarrow \text{This} = \frac{\partial R_2}{\partial P_i}\left[\frac{\partial U}{\partial R_2}\right]_0$$

$$\vdots$$

$$+ \frac{\partial R_m}{\partial P_i}\left(\underline{1m}\,P_1 + \underline{2m}\,P_2 + \cdots + \underline{mm}\,P_m \;;\; + \underline{1m}\,R_1 + \underline{2m}\,R_2 + \cdots + 2\underline{mm}\,R_m\right) \leftarrow \text{This} = \frac{\partial R_m}{\partial P_i}\left[\frac{\partial U}{\partial R_m}\right]_0 .$$

Inasmuch as the deflection of one redundant member depends upon the deflections of other redundant members, the corresponding forces are dependent. When we consider $\frac{\partial U}{\partial R_i}$ we ought to consider $\frac{\partial R_i}{\partial R_i}$. Following the pattern of $\frac{\partial U}{\partial P_i}$,

$$0 = \frac{\partial U}{\partial R_i} = \frac{\partial R_1}{\partial R_i}\left[\frac{\partial U}{\partial R_1}\right]_0 + \frac{\partial R_2}{\partial R_i}\left[\frac{\partial U}{\partial R_2}\right]_0 + \cdots + \frac{\partial R_m}{\partial R_i}\left[\frac{\partial U}{\partial R_m}\right]_0 .$$

Now, if we multiply this expression by $\frac{\partial R_i}{\partial P_i}$, we have

$$0 = \frac{\partial U}{\partial P_i} - \left[\frac{\partial U}{\partial P_i}\right]_0 \;,\; \text{which was to be proved.}$$

To see that every term in $\frac{\partial U}{\partial R_i}$ is zero, consider

$$\left.\begin{aligned} 0 = \frac{\partial U}{\partial R_1} &= \left[\frac{\partial U}{\partial R_1}\right]_0 + \frac{\partial R_2}{\partial R_1}\left[\frac{\partial U}{\partial R_2}\right]_0 + \frac{\partial R_3}{\partial R_1}\left[\frac{\partial U}{\partial R_3}\right]_0 \\ 0 = \frac{\partial U}{\partial R_2} &= \frac{\partial R_1}{\partial R_2}\left[\frac{\partial U}{\partial R_1}\right]_0 + \left[\frac{\partial U}{\partial R_2}\right]_0 + \frac{\partial R_3}{\partial R_2}\left[\frac{\partial U}{\partial R_3}\right]_0 \\ 0 = \frac{\partial U}{\partial R_3} &= \frac{\partial R_1}{\partial R_3}\left[\frac{\partial U}{\partial R_1}\right]_0 + \frac{\partial R_2}{\partial R_3}\left[\frac{\partial U}{\partial R_2}\right]_0 + \left[\frac{\partial U}{\partial R_3}\right]_0 \end{aligned}\right\} \quad \begin{aligned} 0 &= x + ay + bz \\ 0 &= \tfrac{1}{a}x + y + cz \\ 0 &= \tfrac{1}{b}x + \tfrac{1}{c}y + z \end{aligned}$$

$$0 = x = y = z$$

Nevertheless, one final doubt remains: If the coefficients of P_i in $\frac{\partial U}{\partial R_i}$ and $\frac{\partial U}{\partial R_i}$ are not the same as the coefficients of P_i in $\left[\frac{\partial U}{\partial P_i}\right]_0$ and $\left[\frac{\partial U}{\partial R_i}\right]_0$, it may be that $\left[\frac{\partial R_i}{\partial P_i}\right]_0 \neq \frac{\partial R_i}{\partial P_i}$.

The coefficient of P_j in $\frac{\partial U}{\partial P_i}$ is $\left\{\underline{\underline{j1}} + \underline{\underline{j1}}\frac{\partial R_1}{\partial P_i} + \underline{\underline{j2}}\frac{\partial R_2}{\partial P_i} + \cdots + \underline{\underline{jm}}\frac{\partial R_m}{\partial P_i}\right\}$; and in $\left[\frac{\partial U}{\partial P_i}\right]_o$ it is $\left\{\underline{\underline{j1}}\right\}$.

The coefficient of P_j in $\frac{\partial U}{\partial R_i}$ is $\left\{\underline{\underline{j1}} + \underline{\underline{j2}}\frac{\partial R_2}{\partial R_1} + \underline{\underline{j3}}\frac{\partial R_3}{\partial R_1} + \cdots + \underline{\underline{jm}}\frac{\partial R_m}{\partial R_1}\right\}$; and in $\left[\frac{\partial U}{\partial R_i}\right]_o$ it is $\left\{\underline{\underline{j1}}\right\}$.

Now subtracting the respective coefficients, we see that the differences cannot both be zero. If the first difference is zero, then $o = \underline{\underline{j1}}\partial R_1 + \underline{\underline{j2}}\partial R_2 + \cdots + \underline{\underline{jm}}\partial R_m$; but if the second difference is zero, $o = \underline{\underline{j2}}\partial R_2 + \cdots + \underline{\underline{jm}}\partial R_m$. Therefore, the (m+n)th order matrix obtained by reduction of the strain energy function by the simplified method cannot be identical (coefficient for coefficient) to the matrix one ought to obtain by the long method. Moreover, we may conclude that none of the coefficients can correspond between matrices because $\frac{\partial^2 U}{\partial P_i \partial R_j} = \frac{\partial^2 U}{\partial R_j \partial P_i}$; i.e., in each matrix there are complimentary coefficients which must both correspond if one corresponds.

$$\frac{\partial U}{\partial P_i} = P_1\left\{\underline{\underline{i1}} + \underline{\underline{11}}\frac{\partial R_1}{\partial P_i} + \underline{\underline{12}}\frac{\partial R_2}{\partial P_i} + \cdots + \underline{\underline{1m}}\frac{\partial R_m}{\partial P_i}\right\}$$

$$+ P_2\left\{\underline{\underline{2i}} + \underline{\underline{21}}\frac{\partial R_1}{\partial P_i} + \underline{\underline{22}}\frac{\partial R_2}{\partial P_i} + \cdots + \underline{\underline{2m}}\frac{\partial R_m}{\partial P_i}\right\} \qquad \alpha_j = \frac{\partial R_j}{\partial P_i}$$

$$\vdots$$

$$+ P_n\left\{\underline{\underline{in}} + \underline{\underline{n1}}\alpha_1 + \underline{\underline{n2}}\alpha_2 + \cdots + \underline{\underline{nm}}\alpha_m\right\}$$

$$+ R_1\left\{\underline{\underline{i1}} + 2\underline{\underline{11}}\alpha_1 + \underline{\underline{12}}\alpha_2 + \cdots + \underline{\underline{1m}}\alpha_m\right\}$$

$$+ R_2\left\{\underline{\underline{i2}} + \underline{\underline{12}}\alpha_1 + 2\underline{\underline{22}}\alpha_2 + \cdots + \underline{\underline{2m}}\alpha_m\right\} \quad \text{Zero.}$$

$$\vdots$$

$$+ R_m\left\{\underline{\underline{im}} + \underline{\underline{1m}}\alpha_1 + \underline{\underline{2m}}\alpha_2 + \cdots + 2\underline{\underline{mm}}\alpha_m\right\}.$$

On the other hand, if the co-factor of every R_j above is zero, it may be seen that the two methods are absolutely identical. The general expression for the cofactors is $\frac{\partial}{\partial P_i}$ of $\left[\frac{\partial U}{\partial R_j}\right]_o$; <u>which certainly is zero.</u>

$$o = \left[\frac{\partial^2 U}{\partial R_j \partial P_i}\right]_o + \left[\frac{\partial^2 U}{\partial R_1 \partial R_j}\right]_o \frac{\partial R_1}{\partial P_i} + \left[\frac{\partial^2 U}{\partial R_2 \partial R_j}\right]_o \frac{\partial R_2}{\partial P_i} + \cdots + \left[\frac{\partial^2 U}{\partial R_m \partial R_j}\right]_o \frac{\partial R_m}{\partial P_i}.$$

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$$a_{11}x_1 + a_{12}x_2 + c_1 = 0 \quad = \quad \frac{\partial U}{\partial R_1} = 0$$
$$a_{21}x_1 + a_{22}x_2 + c_2 = 0 \quad = \quad \frac{\partial U}{\partial R_2} = 0$$

$$\frac{\partial^2 U}{\partial R_1 \partial R_2} = \frac{\partial^2 U}{\partial R_2 \partial R_1}$$

$$\therefore a_{12} = a_{21}$$

$$a_{22}a_{11}x_1 + a_{22}a_{12}x_2 + a_{22}c_1 = 0$$
$$-\left(a_{12}a_{21}x_1 - a_{12}a_{22}x_2 - a_{12}c_2 = 0\right)$$

$$x_1(a_{22}a_{11} - a_{12}a_{21}) = a_{12}c_2 - a_{22}c_1$$

$$x_1 = \frac{a_{12}c_2 - a_{22}c_1}{a_{22}a_{11} - a_{12}a_{21}} \qquad x_2 = \frac{a_{11}c_2 - a_{21}c_1}{a_{21}a_{12} - a_{11}a_{22}}$$

Let $\beta = a_{11}a_{22} - a_{12}a_{21}$

$$\therefore x_1 = \frac{a_{12}}{\beta}c_2 - \frac{a_{22}}{\beta}c_1$$

$$x_2 = \frac{a_{21}}{a_{11}a_{22} - a_{12}a_{21}}c_1 - \frac{a_{11}}{a_{11}a_{22} - a_{12}a_{21}}c_2$$

$$x_1 = \frac{a_{12}}{\beta}c_2 - \frac{a_{22}}{\beta}c_1$$
$$x_2 = -\frac{a_{21}}{\beta}c_2 + \frac{a_{11}}{\beta}c_1$$

Let $c_1 = b_{11}P_1 + b_{12}P_2 + \cdots b_{1n}P_n$
$c_2 = b_{21}P_1 + b_{22}P_2 + \cdots b_{2n}P_n$

$$x_1 = -\frac{a_{22}}{\beta}b_{11}P_1 - \frac{a_{22}}{\beta}b_{12}P_2 - \cdots \frac{a_{22}}{\beta}b_{1n}P_n + \frac{a_{12}}{\beta}b_{21}P_1 + \frac{a_{12}}{\beta}b_{22}P_2 + \cdots + \frac{a_{12}}{\beta}b_{2n}P_n$$
$$x_2 = \frac{a_{21}}{\beta}b_{11}P_1 + \frac{a_{21}}{\beta}b_{12}P_2 + \cdots + \frac{a_{21}}{\beta}b_{1n}P_n - \frac{a_{11}}{\beta}b_{21}P_1 - \frac{a_{11}}{\beta}b_{22}P_2 - \cdots - \frac{a_{11}}{\beta}b_{2n}P_n$$

Theorm: $\sum_{k=1}^{k=K} \frac{\partial^2 U}{\partial R_k \partial P_i}\frac{\partial R_k}{\partial P_j} = \sum_{k=1}^{k=K} \frac{\partial^2 U}{\partial R_k \partial P_j}\frac{\partial R_k}{\partial P_i}$

$$b_{11}\left[\frac{a_{12}}{\beta}b_{22} - \frac{a_{22}}{\beta}b_{12}\right] + b_{21}\left[\frac{a_{21}}{\beta}b_{12} - \frac{a_{11}}{\beta}b_{22}\right] = b_{12}\left[\frac{a_{12}}{\beta}b_{21} - \frac{a_{22}}{\beta}b_{11}\right] + b_{22}\left[\frac{a_{21}}{\beta}b_{11} - \frac{a_{11}}{\beta}b_{21}\right]$$

$$a_{12}b_{11}b_{22} - a_{22}b_{11}b_{12} + a_{21}b_{12}b_{21} - a_{11}b_{21}b_{22} = a_{12}b_{12}b_{21} - a_{22}b_{11}b_{12} + a_{21}b_{11}b_{22} - a_{11}b_{21}b_{22}$$

Inasmuch as $a_{12} = a_{21}$, we see that the check theorm is trivial.

M.

AVRO AIRCRAFT LIMITED

TECHNICAL DEPARTMENT (Aircraft)

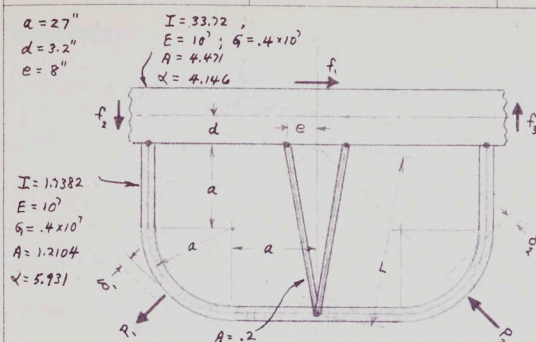
REPORT No.	7-0558-8
SHEET No.	C1
AIRCRAFT	
Deflection Formulas Check Analysis	
PREPARED BY	V.J. Maai
DATE	4-4-55
CHECKED BY	
DATE	

$a = 27''$
$d = 3.2''$
$e = 8''$

$I = 33.72$, $E = 10^7$; $G = .4 \times 10^7$, $A = 4.471$, $\alpha = 4.146$

$I = 1.7382$, $E = 10^7$, $G = .4 \times 10^7$, $A = 1.2104$, $\alpha = 5.931$

$A = .2$, $E = 2.9 \times 10^7$



Note: $P_2 = P_1$

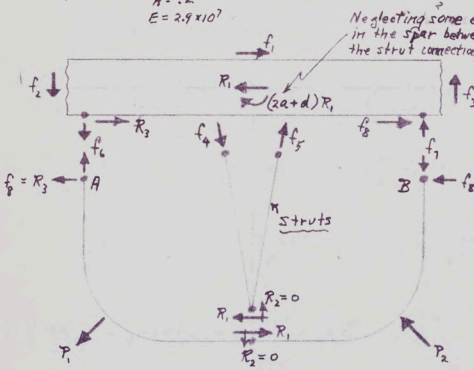
By statics, $f_2 = f_3$

$$f_1 = \frac{P_1}{\sqrt{2}} + \frac{P_2}{\sqrt{2}}$$

$$f_2 + f_3 = \frac{2d\frac{P_1}{\sqrt{2}} + 2d\frac{P_2}{\sqrt{2}}}{2a}$$

$$\therefore f_2 = f_3 = \frac{P_1 d}{2\sqrt{2}a} + \frac{P_2 d}{2\sqrt{2}a} = \frac{d}{2a} f_1$$

Neglecting some energy in the spar between the strut connections.



$R_2 = 0$.

$$f_4 = f_5 = \frac{R_1 L}{2e}$$

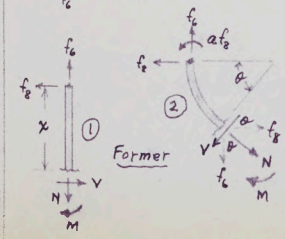
$$f_6 = f_7 = \frac{R_1}{2} \quad (M_A = 0) \quad (M_B = 0)$$

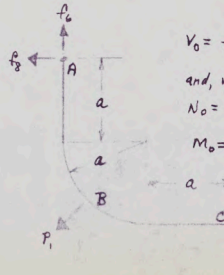
$$f_8 = R_3 = \frac{R_1}{2} - \frac{P_1}{2\sqrt{2}} - \frac{P_2}{2\sqrt{2}} = \frac{R_1}{2} - \frac{f_1}{2}$$

$$f_2 + f_6 = \frac{d}{2a} f_1 + \frac{R_1}{2} = f_9$$

$$f_3 + f_7 = \frac{d}{2a} f_1 + \frac{R_1}{2} = f_{10}$$

Spar:





$$V_0 = f_6 - \frac{P_1}{\sqrt{2}} = \frac{R_1}{2} - \frac{P_1}{\sqrt{2}}$$

and, when $P_1 = P_2$, $V_0 = f_8$.

$$N_0 = f_8 + \frac{P_1}{\sqrt{2}} = f_6$$

$$M_0 = 2af_8 - 2af_6 + a\sqrt{2}P_1 = a\{-f_1 + \sqrt{2}P_1\} = 0$$

$$\therefore U_{AB} = U_{BC}$$

Strut Energy:

$$U_{ST} = 2\frac{f_4^2 L}{2R_{ST}E_{ST}} = \frac{R_1^2 L^3}{4e^2 A_{ST} E_{ST}} = \underline{1} R_1^2$$

$$\underline{1} = \frac{L^3}{4e^2 A_{ST} E_{ST}} .$$

Spar Energy:

$$U_{SP} = 2\int_0^{2a}\left\{\frac{M^2}{2E_{SP}I_{SP}} + \frac{N^2}{2A_{SP}E_{SP}} + \frac{\alpha_{SP}V^2}{2A_{SP}G_{SP}}\right\} dx$$

$$M = f_9 x + d f_8 \quad ; \quad N = f_8 \quad ; \quad V = f_9 .$$

$$U_{SP} = 2\int_0^{2a}\left\{\frac{f_9^2 x^2 + 2d f_8 f_9 x + d^2 f_8^2}{2E_{SP}I_{SP}} + \frac{f_8^2}{2A_{SP}E_{SP}} + \frac{\alpha_{SP} f_9^2}{2A_{SP}G_{SP}}\right\} dx$$

$$= \frac{1}{E_{SP}I_{SP}}\left(\frac{8a^3}{3} f_9^2 + 4a^2 d f_8 f_9 + 2a d^2 f_8^2\right) + \frac{2a}{A_{SP}E_{SP}} f_8^2 + \frac{2a\alpha_{SP}}{A_{SP}G_{SP}} f_9^2$$

$$\underline{2} = \frac{2a}{E_{SP}I_{SP}} \quad ; \quad \underline{3} = \frac{2a}{A_{SP}E_{SP}} \quad ; \quad \underline{4} = \frac{2a\alpha_{SP}}{A_{SP}G_{SP}}$$

$$U_{SP} = \underline{2}\frac{4a^2}{3} f_9^2 + \underline{2} 2ad f_8 f_9 + \underline{2} d^2 f_8^2 + \underline{3} f_8^2 + \underline{4} f_9^2$$

$$\underline{5} = \underline{2}\frac{4a^2}{3} + \underline{4} \quad ; \quad \underline{6} = \underline{2} d^2 + \underline{3} \quad ; \quad \underline{7} = \underline{2} 2ad$$

$$U_{SP} = \underline{6} f_8^2 + \underline{7} f_8 f_9 + \underline{5} f_9^2$$

Former Energy:

$U_{FO_{AB}}$: $\quad V = f_8 \quad ; \quad N = f_6 \quad ; \quad M = f_8 x$

$$U_{FO_{AB}} = \int_0^a = \frac{f_8^2 a^3}{6E_{FO}I_{FO}} + \frac{f_6^2 a}{2A_{FO}E_{FO}} + \frac{\alpha_{FO} f_8^2 a}{2A_{FO}G_{FO}}$$

$$\underline{8} = \frac{a^3}{6E_{FO}I_{FO}} \quad ; \quad \underline{9} = \frac{a}{2A_{FO}E_{FO}} \quad ; \quad \underline{10} = \frac{\alpha_{FO} a}{2A_{FO}G_{FO}} \quad ; \quad \underline{11} = \underline{8} + \underline{10}$$

$$U_{FO_{AB}} = \underline{9} f_6^2 + \underline{11} f_8^2$$

FORM 1198A

$U_{FO_{AB_2}}$:

$N = f_8 \sin\theta + f_6 \cos\theta$ $\qquad f_8 - f_6 = -\frac{f_1}{2}$

$V = f_6 \sin\theta - f_8 \cos\theta$ $\qquad ds = a\,d\theta$

$M = a f_8 + f_8 a \sin\theta - f_6 a(1-\cos\theta) = a\left(\left[f_8 - f_6\right] + f_8 \sin\theta + f_6 \cos\theta\right)$

$$U_{FO_{AB_2}} = \int_0^{\pi/4} \left\{ 3\,\underline{8}\left(\frac{f_1^2}{4} + f_8^2 \sin^2\theta + f_6^2 \cos^2\theta - f_1 f_8 \sin\theta - f_1 f_6 \cos\theta + f_6 f_8 \sin 2\theta\right) + \underline{9}\left(f_8^2 \sin^2\theta + f_6^2 \cos^2\theta + f_6 f_8 \sin 2\theta\right) + \underline{10}\left(f_6^2 \sin^2\theta + f_8^2 \cos^2\theta - f_6 f_8 \sin 2\theta\right) \right\} d\theta$$

$\underline{12} = \int_0^{\pi/4} \sin^2\theta\, d\theta = \left[\frac{\theta}{2} - \frac{\sin 2\theta}{4}\right]_0^{\pi/4} = .14270$

$\underline{13} = \int_0^{\pi/4} \cos^2\theta\, d\theta = \left[\frac{\theta}{2} + \frac{\sin 2\theta}{4}\right]_0^{\pi/4} = .64270$

$\underline{14} = \int_0^{\pi/4} \sin\theta\, d\theta = \left[-\cos\theta\right]_0^{\pi/4} = .292893$

$\underline{15} = \int_0^{\pi/4} \cos\theta\, d\theta = \left[\sin\theta\right]_0^{\pi/4} = .707107$

$\underline{16} = \int_0^{\pi/4} \sin 2\theta\, d\theta = \left[-\frac{1}{2}\cos 2\theta\right]_0^{\pi/4} = .50000$

$$U_{FO_{AB_2}} = \overbrace{.75 \times .7854 \times \underline{8}}^{\underline{17}} f_1^2 + 3\,\underline{8}\,\underline{\overset{18}{12}}\, f_8^2 + 3\,\underline{8}\,\underline{\overset{19}{13}}\, f_6^2 - 3\,\underline{8}\,\underline{\overset{-20}{14}}\, f_1 f_8 - 3\,\underline{8}\,\underline{\overset{-21}{15}}\, f_1 f_6 + 3\,\underline{8}\,\underline{\overset{22}{16}}\, f_6 f_8 + \underline{9}\,\underline{\overset{23}{12}}\, f_8^2 + \underline{9}\,\underline{\overset{24}{13}}\, f_6^2 + \underline{9}\,\underline{\overset{25}{16}}\, f_6 f_8 + \underline{10}\,\underline{\overset{26}{12}}\, f_6^2 + \underline{10}\,\underline{\overset{27}{13}}\, f_8^2 - \underline{10}\,\underline{\overset{-28}{16}}\, f_6 f_8$$

$\underline{29} = \underline{19} + \underline{24} + \underline{26}$ $\qquad \underline{32} = \underline{9} + \underline{29}$

$\underline{30} = \underline{18} + \underline{23} + \underline{27}$ $\qquad \underline{33} = \underline{11} + \underline{30}$

$\underline{31} = \underline{22} + \underline{25} - \underline{28}$

$$U_{FO_{AB_2}} = \underline{17} f_1^2 + \underline{29} f_6^2 + \underline{30} f_8^2 - \underline{21} f_1 f_6 - \underline{20} f_1 f_8 + \underline{31} f_6 f_8$$

$$U_{FO_{AB}} = \underline{17} f_1^2 + \underline{32} f_6^2 + \underline{33} f_8^2 - \underline{21} f_1 f_6 - \underline{20} f_1 f_8 + \underline{31} f_6 f_8$$

Inasmuch as $f_6 = f_7 = \frac{R_1}{2}$; $R_2 = f_8 = \frac{R_1}{2} - \frac{f_1}{2}$; and $P_1 = P_2$; the strain energy of the right half equals the energy of the left half. Therefore $U_{FO} = 4\,U_{FO_{AB}}$.

$\underline{34} = \underline{6} + 4\underline{33}$

$$U = \underline{1} R_1^2 + 4\underline{17} f_1^2 + 4\underline{32} f_6^2 + \underline{34} f_8^2 + \underline{5} f_9^2 - 4\underline{21} f_1 f_6 - 4\underline{20} f_1 f_8 + 4\underline{31} f_6 f_8 + \underline{7} f_8 f_9 .$$

$\frac{\partial U}{\partial R_1} = 0$; $\frac{\partial}{\partial P_1}\left(\frac{\partial U}{\partial R_1}\right) = 0$; $\frac{\partial}{\partial P_2}\left(\frac{\partial U}{\partial R_1}\right) = 0$. These yield $P_1 = \frac{\partial R_1}{\partial P_1} P_1 + \frac{\partial R_1}{\partial P_2} P_2$.

$\frac{\partial U}{\partial P_1} = \delta_1$; $\frac{\partial \delta_1}{\partial P_1} = \frac{\partial^2 U}{\partial P_1^2} + \frac{\partial^2 U}{\partial P_1 \partial R_1}\frac{\partial R_1}{\partial P_1}$; $\frac{\partial \delta_1}{\partial P_2} = \frac{\partial^2 U}{\partial P_1 \partial P_2} + \frac{\partial^2 U}{\partial P_1 \partial R_1}\frac{\partial R_1}{\partial P_2}$

These yield $\delta_1 = \frac{\partial \delta_1}{\partial P_1} P_1 + \frac{\partial \delta_1}{\partial P_2} P_2$.

$\frac{\partial U}{\partial P_2} = \delta_2$; $\frac{\partial \delta_2}{\partial P_1} = \frac{\partial \delta_1}{\partial P_2}$; $\frac{\partial \delta_2}{\partial P_2} = \frac{\partial^2 U}{\partial P_2^2} + \frac{\partial^2 U}{\partial P_2 \partial R_1}\frac{\partial R_1}{\partial P_2}$.

and, $\delta_2 = \frac{\partial \delta_2}{\partial P_1} P_1 + \frac{\partial \delta_2}{\partial P_2} P_2$.

When we differentiate the strain energy function,

for $\frac{\partial U}{\partial R_1}$, then $\frac{\partial P}{\partial R} = 0$, and $\frac{\partial f}{\partial R} = \frac{\partial f}{\partial R}$;

for $\frac{\partial U}{\partial P}$, then $\frac{\partial R}{\partial P} = 0$, and $\frac{\partial f}{\partial P} = \frac{\partial f}{\partial P}$;

} This also for $\frac{\partial^2 U}{\partial R_1^2}$, $\frac{\partial^2 U}{\partial P_1^2}$, and $\frac{\partial U}{\partial R_1 \partial P_1}$

for $\frac{\partial}{\partial P}\left(\frac{\partial U}{\partial R}\right)$, then $\frac{\partial R}{\partial P} = \frac{\partial R}{\partial P}$

Note: for only two applied forces and one redundant this method is not the shortest that we might use. We use it now to check a more complicated problem.

$$\frac{\partial U}{\partial X} = 2\underline{1}\,R_1\frac{\partial R_1}{\partial X} + 8\underline{17}\,f_1\frac{\partial f_1}{\partial X} + 8\underline{32}\,f_6\frac{\partial f_6}{\partial X} + 2\underline{34}\,f_8\frac{\partial f_8}{\partial X} + 2\underline{5}\,f_9\frac{\partial f_9}{\partial X} - 4\underline{21}\,f_1\frac{\partial f_6}{\partial X} - 4\underline{21}\,f_6\frac{\partial f_1}{\partial X}$$
$$- 4\underline{20}\,f_1\frac{\partial f_8}{\partial X} - 4\underline{20}\,f_8\frac{\partial f_1}{\partial X} + 4\underline{31}\,f_6\frac{\partial f_8}{\partial X} + 4\underline{21}\,f_8\frac{\partial f_6}{\partial X} + \underline{7}\,f_8\frac{\partial f_9}{\partial X} + \underline{7}\,f_9\frac{\partial f_8}{\partial X}.$$

Note: $\frac{\partial f}{\partial X}$ are constants. Therefore $\frac{\partial}{\partial Y}\left(\frac{\partial f}{\partial X}\right) = 0$. Also $\frac{\partial}{\partial Y}\left(\frac{\partial R}{\partial X}\right) = 0$.

$$\frac{\partial^2 U}{\partial X \partial Y} = 2\underline{1}\frac{\partial R_1}{\partial Y}\frac{\partial R_1}{\partial X} + 8\underline{17}\frac{\partial f_1}{\partial Y}\frac{\partial f_1}{\partial X} + 8\underline{32}\frac{\partial f_6}{\partial Y}\frac{\partial f_6}{\partial X} + 2\underline{34}\frac{\partial f_8}{\partial Y}\frac{\partial f_8}{\partial X} + 2\underline{5}\frac{\partial f_9}{\partial Y}\frac{\partial f_9}{\partial X} - 4\underline{21}\frac{\partial f_1}{\partial Y}\frac{\partial f_6}{\partial X}$$
$$- 4\underline{21}\frac{\partial f_6}{\partial Y}\frac{\partial f_1}{\partial X} - 4\underline{20}\frac{\partial f_1}{\partial Y}\frac{\partial f_8}{\partial X} - 4\underline{20}\frac{\partial f_8}{\partial Y}\frac{\partial f_1}{\partial X} + 4\underline{31}\frac{\partial f_6}{\partial Y}\frac{\partial f_8}{\partial X} + 4\underline{21}\frac{\partial f_8}{\partial Y}\frac{\partial f_6}{\partial X} + 7\frac{\partial f_8}{\partial Y}\frac{\partial f_9}{\partial X} + 7\frac{\partial f_9}{\partial Y}\frac{\partial f_8}{\partial X}.$$

$$\frac{\partial}{\partial P_1}\left(\frac{\partial U}{\partial R_1}\right) = \frac{\partial^2 U}{\partial P_1 \partial R_1} + \frac{\partial^2 U}{\partial R_1^2}\frac{\partial R_1}{\partial P_1} = 0$$

$$\frac{\partial}{\partial P_2}\left(\frac{\partial U}{\partial R_1}\right) = \frac{\partial^2 U}{\partial P_2 \partial R_1} + \frac{\partial^2 U}{\partial R_1^2}\frac{\partial R_1}{\partial P_2} = 0$$

For substitution in $\frac{\partial^2 U}{\partial x \partial y}$,

$\frac{\partial R_1}{\partial R_1} = 1$ $\quad \frac{\partial R_1}{\partial P_1} = 0$ $\quad \frac{\partial R_1}{\partial P_2} = 0$

$\frac{\partial f_1}{\partial R_1} = 0$ $\quad \frac{\partial f_1}{\partial P_1} = \frac{1}{\sqrt{2}}$ $\quad \frac{\partial f_1}{\partial P_2} = \frac{1}{\sqrt{2}}$

$\frac{\partial f_6}{\partial R_1} = \frac{1}{2}$ $\quad \frac{\partial f_6}{\partial P_1} = 0$ $\quad \frac{\partial f_6}{\partial P_2} = 0$

$\frac{\partial f_8}{\partial R_1} = \frac{1}{2}$ $\quad \frac{\partial f_8}{\partial P_1} = -\frac{1}{2\sqrt{2}}$ $\quad \frac{\partial f_8}{\partial P_2} = -\frac{1}{2\sqrt{2}}$

$\frac{\partial f_9}{\partial R_1} = \frac{1}{2}$ $\quad \frac{\partial f_9}{\partial P_1} = \frac{d}{a}\frac{1}{2\sqrt{2}}$ $\quad \frac{\partial f_9}{\partial P_2} = \frac{d}{a}\frac{1}{2\sqrt{2}}$

$$\frac{\partial^2 U}{\partial P_1^2} = 8\,\underline{17}\,\frac{1}{2} + 2\,\underline{34}\,\frac{1}{8} + 2\,\underline{5}\,\frac{d^2}{a^2}\frac{1}{8} + 4\,\underline{20}\,\frac{1}{4} + 4\,\underline{20}\,\frac{1}{4} - 7\,\frac{d}{a}\frac{1}{8} - 7\,\frac{d}{a}\frac{1}{8}$$

$$= \underline{4\,17 + .25\,34 + .25\frac{d^2}{a^2}5 + 2\,20 - .25\frac{d}{a}7}\ .$$

$\frac{\partial^2 U}{\partial P_1 \partial P_2} = \frac{\partial^2 U}{\partial P_1^2}$ because all $\frac{\partial f}{\partial P_1} = \frac{\partial f}{\partial P_2}$

$$\frac{\partial^2 U}{\partial P_1 \partial R_1} = -2\,\underline{34}\,\frac{1}{4\sqrt{2}} + 2\,\underline{5}\,\frac{d}{a}\frac{1}{4\sqrt{2}} - 4\,\underline{21}\,\frac{1}{2\sqrt{2}} - 4\,\underline{20}\,\frac{1}{2\sqrt{2}} - 4\,\underline{31}\,\frac{1}{4\sqrt{2}} - 7\,\frac{1}{4\sqrt{2}} + 7\,\frac{d}{a}\frac{1}{4\sqrt{2}}$$

$$= \underline{.707107\left(-.5\,34 + .5\frac{d}{a}5 - 2\,21 - 2\,20 - 31 - .25\,7 + .25\frac{d}{a}7\right)}$$

$\frac{\partial^2 U}{\partial P_2^2} = \frac{\partial^2 U}{\partial P_1^2}$, because all $\frac{\partial f}{\partial P_1} = \frac{\partial f}{\partial P_2}$

$\frac{\partial^2 U}{\partial P_2 \partial P_1} = \frac{\partial^2 U}{\partial P_1 \partial P_2}$, a must.

$\frac{\partial^2 U}{\partial P_2 \partial R_1} = \frac{\partial^2 U}{\partial P_1 \partial R_1}$, because all $\frac{\partial f}{\partial P_1} = \frac{\partial f}{\partial P_2}$

$$\frac{\partial^2 U}{\partial R_1^2} = 2\,\underline{1} + 8\,\underline{32}\,\frac{1}{4} + 2\,\underline{34}\,\frac{1}{4} + 2\,\underline{5}\,\frac{1}{4} + 4\,\underline{31}\,\frac{1}{4} + 4\,\underline{31}\,\frac{1}{4} + 7\,\frac{1}{4} + 7\,\frac{1}{4}$$

$$= \underline{2\,1 + 2\,32 + .5\,34 + .5\,5 + 2\,31 + .5\,7}\ .$$

$$\frac{\partial R_1}{\partial P_1} = -\frac{\frac{\partial^2 U}{\partial P_1 \partial R_1}}{\frac{\partial^2 U}{\partial R_1^2}} \qquad R_1 = 2\frac{\partial R_2}{\partial P_1}P_1$$

$$\frac{\partial R_1}{\partial P_2} = -\frac{\frac{\partial^2 U}{\partial P_2 \partial R_1}}{\frac{\partial^2 U}{\partial R_1^2}} = \frac{\partial R_1}{\partial P_1}\ . \qquad \delta_1 = \delta_2 = 2\frac{\partial \delta_1}{\partial P_1}P_1 = 2P_1\left[\frac{\partial^2 U}{\partial P_1^2} + \frac{\partial^2 U}{\partial P_1 \partial R_1}\frac{\partial R_1}{\partial P_1}\right]$$

AVRO AIRCRAFT LIMITED

TECHNICAL DEPARTMENT (Aircraft)

AIRCRAFT		REPORT No. 7-0558-8	
		SHEET No. C6	
		PREPARED BY	DATE
		V.J. Mass	4-14-55
		CHECKED BY	DATE

Computations: $q = 10^{-7}$

1 = $\frac{(54.5894)^3}{4 \times 64 \times .2 \times 10^3 \times 2.9}$ = 1095.61 q

2 = $\frac{54}{10^7 \times 3.3072}$ = 1.60142 q

3 = $\frac{54}{4.471 \times 10^3}$ = 12.0778 q

4 = $\frac{54 \times 4.146}{4.471 \times .4 \times 10^7}$ = 125.1865 q

5 = 1.60142 q × 1.33333 × 27^2 + 125.1865 q = 1681.767 q

6 = 1.60142 × 3.2^2 q + 12.0778 q = 28.4763 q
16.3985

7 = 1.60142 q × 54 × 3.2 = 276.722 q

8 = $\frac{27^3}{6 \times 10^7 \times 1.7282}$ = 1887.2972 q

9 = $\frac{27}{2 \times 1.2104 \times 10^3}$ = 11.1533 q

10 = $\frac{5.931 \times 27}{2 \times 1.2104 \times .4 \times 10^3}$ = 165.3755 q

11 = 1887.2972 q + 165.3755 q = 2052.673 q

12 = .14270

13 = .64270

14 = .29289

15 = .70711

16 = .50000

17 = .75 × .78540 × 1887.297 q = 1111.712 q

18 = 3 × 1887.297 q × .14270 = 807.9519 q

19 = 3 × 1887.297 q × .64270 = 3638.897 q

20 = 3 × 1887.297 q × .29289 = 1658.311 q

21 = 3 × 1887.297 q × .70711 = 4003.580 q

22 = 3 × 1887.297 q × .50000 = 2830.9455 q

FORM 1104

$\underline{23} = 11.1533\ q \times .14270 = \underline{1.5916}\ q$
$\underline{24} = 11.1533\ q \times .64270 = \underline{7.1682}\ q$
$\underline{25} = 11.1533\ q \times .50000 = \underline{5.5766}\ q$

$\underline{26} = 165.3755\ q \times .14270 = \underline{23.5991}\ q$
$\underline{27} = 165.3755\ q \times .64270 = \underline{106.2868}\ q$
$\underline{28} = 165.3755\ q \times .50000 = \underline{82.6878}\ q$

$\underline{29} = 3638.897\ q + 7.1682\ q + 23.5991\ q = \underline{3669.6643}\ q$
$\underline{30} = 807.8519\ q + 1.5916\ q + 106.2868\ q = \underline{915.8303}\ q$
$\underline{31} = 2830.9455\ q + 5.5766\ q - 82.6878\ q = \underline{2753.8343}\ q$

$\underline{32} = 11.1533\ q + 3669.6643\ q = \underline{3680.8176}\ q$
$\underline{33} = 2052.673\ q + 915.8303\ q = \underline{2968.5033}\ q$

$\underline{34} = 28.4763\ q + 4 \times 2968.5033\ q = \underline{11,902.4895}\ q$

$$\frac{\partial^2 U}{\partial R_1^2} = 4 \times 1111.712\ q + .25 \times 11,902.4895\ q + .25 \times \left(\frac{3.2}{27}\right)^2 \times 1681.767\ q + 2 \times 1658.311\ q - .25 \times \frac{3.2}{27} \times 276.722\ q$$

$$= q\,(4446.848 + 2975.622 + 5.904 + 3316.622 - 8.199)$$

$$= \underline{.001073680}$$

$$\frac{\partial^2 U}{\partial R_1^2} = q\,(2 \times 1095.61 + 2 \times 3680.8176 + .5 \times 11,902.4895 + .5 \times 1681.767 + 2 \times 2753.8343 + .5 \times 276.722)$$

$$= q\,(2191.220 + 7361.635 + 5951.245 + 840.884 + 5507.669 + 138.361)$$

$$= \underline{.002199105}$$

$$\frac{\partial^2 U}{\partial R_1 \partial R_1} = .70711\,q\,(-.5 \times 11,902.4895 + .5\,\frac{3.2}{27} \times 1681.767 - 2 \times 4003.580 - 2 \times 1658.311 - 2753.8343 - .25 \times 276.722 + .25 \times \frac{3.2}{27} \times 276.722)$$

$$= .70711\,q\,(-5951.245 + 99.660 - 8007.160 - 3316.622 - 2753.8343 - 69.181 + 8.199)$$

$$= .70711\,q\,(-19,990.183)$$

$$= \underline{-.0014135258}$$

$$\frac{\partial R_1}{\partial P_1} = -\frac{-.00141352\overline{58}}{.00219910\overline{5}} = +.642773$$

$$\delta_1 = 2P_1\left[.00107368\overline{0} + (-.00141352\overline{6})(+.642773) + .00016510\overline{4}\right] \quad (-.00090857\overline{62})$$

$$\delta_1 = .00033021\,P$$
$$\delta_2 = .00033021\,P$$

Note: Inherently $P_1 = P_2 = P$; i.e., no other condition can be considered in this set-up in which $R_2 = 0$, and $R_3 = f_8$.

$$\frac{P}{\delta} = 3028.\ \#/\text{inch}.$$

Now, if we return to the strain energy function and substitute P for P_1 and for P_2 before we differentiate, and then say that $\frac{\partial U}{\partial P} = 2\delta$, because $\partial U = \delta_1 \partial P_1 + \delta_2 \partial P_2$

$$\frac{\partial f_1}{\partial P} = \sqrt{2} \quad ; \quad \frac{\partial f_6}{\partial P} = 0 \quad ; \quad \frac{\partial f_8}{\partial P} = -\frac{1}{\sqrt{2}} \quad ; \quad \frac{\partial f_9}{\partial P} = \frac{d}{2a}\sqrt{2} = \frac{d}{a\sqrt{2}}$$

$$\frac{\partial f_1}{\partial R_1} = 0 \quad ; \quad \frac{1}{2} = \frac{\partial f_6}{\partial R} = \frac{\partial f_8}{\partial R} = \frac{\partial f_9}{\partial R} \quad \text{as before.}$$

$$\frac{\partial R}{\partial P} = -\frac{\frac{\partial^2 U}{\partial R P}}{\frac{\partial^2 U}{\partial R_1^2}} \quad \text{as before, but}$$

$$\delta = \frac{P}{2}\left(\frac{\partial^2 U}{\partial P_1^2} + \frac{\partial^2 U}{\partial R P}\frac{\partial R}{\partial P}\right)$$

$$\frac{\partial^2 U}{\partial P^2} = 8\,\underline{17}\,2 + 2\,\underline{34}\,\frac{1}{2} + 2\,\underline{5}\,\frac{d^2}{2a^2} + 4\,\underline{20} + 4\,\underline{20} - 7\,\frac{d}{2a} - 7\,\frac{d}{2a}$$

$$= 16 \times 1111.312\,g + 11{,}902.4895\,g + \left(\frac{3.2}{27}\right)^2 \times 1681.767\,g + 8 \times 1658.311\,g - 276.722\,g\,\frac{3.2}{27}$$

$$= g\,(17{,}787.392 + 11{,}902.490 + 23.623 + 13{,}266.488 - 32.797)$$

$$= .004294719 \ .$$

$$\frac{\partial^2 U}{\partial R^2} = .00219910\overline{5} \quad \text{as before.}$$

$$\frac{\partial^2 U}{\partial P R} = 2\,\underline{34}\left(-\frac{1}{2\sqrt{2}}\right) + 2\,\underline{5}\,\frac{d}{a}\,\frac{1}{2\sqrt{2}} - 4\,\underline{21}\,\frac{1}{\sqrt{2}} - 4\,\underline{20}\,\frac{1}{\sqrt{2}} - 4\,\underline{31}\,\frac{1}{2\sqrt{2}} - 7\,\frac{1}{2\sqrt{2}} + 7\,\frac{d}{a}\,\frac{1}{2\sqrt{2}}$$

$$= .70711\,g\,(-11{,}902.4895 + 199.320 - 16{,}014.320 - 6633.244 - 5507.669 - 138.361 + 16.398 - 39{,}980.366)$$

$$= -.00282705\overline{17}$$

$$\frac{\partial R}{\partial P} = -\frac{-.0028270517}{.002199105} = 1.285546$$

$$\delta = \frac{P}{2}\left(.004294719 - \frac{-.0036344305}{.000660414} - .0028270517 \times 1.285546\right)$$

$$\delta = .000033021\,P$$

$$\frac{P}{\delta} = 3028 \text{ \#/IN.} \quad \text{as before.}$$

Note:
The result obtained in "Stiffness of Former-Spar-Strut Assembly" of 2-4-55 was $P/\delta = 2947$ #/IN. The difference is attributed to the difference in type of spar reactions.

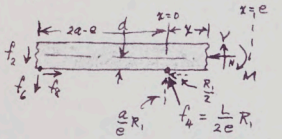
If we had not treated forces f_4 and f_5 as a pure moment acting at the midspan of the spar, would our answer have been much different?

$$U_{SP} = 2\int_0^{2a-e} + 2\int_{2a-e}^{2a} = U_{SP_1} + U_{SP_2}. \qquad \text{Say } \frac{2a-e}{2a} = \underline{35}$$

$$U_{SP_1} = \underline{35}^3(\underline{5}-\underline{4})f_9^2 + \underline{35}^2\,\underline{7}\,f_8 f_9 + \underline{35}(\underline{6}-\underline{3})f_8^2 + \underline{35}\,\underline{3}\,f_8^2 + \underline{35}\,\underline{4}\,f_9^2$$

$$\underline{36} = \underline{35}\,\underline{6}\;; \quad \underline{37} = \underline{35}^2\,\underline{7}\;; \quad \underline{38} = \underline{35}^3(\underline{5}-\underline{4}) + \underline{35}\,\underline{4}$$

$$U_{SP_1} = \underline{36}\,f_8^2 + \underline{37}\,f_8 f_9 + \underline{38}\,f_9^2$$



$$N = f_8 - \frac{R_1}{2} = -\frac{f_1}{2}$$

$$V = f_9 - \frac{d}{e}R_1 = \frac{d}{2a}f_1 - R_1\left(\frac{d}{e}-\frac{1}{2}\right) \qquad \underline{35}\,\frac{d}{e}$$

$$M = f_9(\underline{39}\,2a-e) + f_8 d + f_9 x - \frac{R_1}{2}d - \frac{d}{e}R_1 x$$

$$= \underline{39}\,\frac{d}{2a}f_1 + \underline{39}\,\frac{R_1}{2} + d\frac{R_1}{2} - d\frac{f_1}{2} - d\frac{R_1}{2} + \left(f_9 - \frac{d}{e}R_1\right)x$$

$$= f_1\left(\underline{39}\,\frac{d}{2a} - \frac{d}{2}\right) + \underline{39}\,\frac{R_1}{2} + Vx \qquad (\underline{44} = \underline{39}\,\tfrac{d}{2a} - \tfrac{d}{2})$$

$$U_{SP_2} = \chi\int_0^e \left\{\frac{u^2 + 2uVx + V^2x^2}{2E_{SP}I_{SP}} + \frac{N^2}{2A_{SP}E_{SP}} + \frac{\kappa_{SP}V^2}{2A_{SP}G_{SP}}\right\}dx \qquad (u = \underline{44}\,f_1 + \underline{39}\,\tfrac{R_1}{2})$$

$$\underline{40} = \frac{e}{2a}\;; \quad \underline{41} = \underline{40}\,\underline{2}\;; \quad \underline{42} = \underline{40}\,\underline{3}\;; \quad \underline{43} = \underline{40}\,\underline{4}\;; \quad \underline{44} = \frac{d}{2}\left(\frac{\underline{39}}{2}-1\right)$$

$$U_{SP_2} = \underline{41}\,u^2 + \underline{41}\,euV + \underline{41}\,\frac{e^2}{3}V^2 + \underline{42}\,N^2 + \underline{43}\,V^2. \qquad \underline{45} = \underline{41}\,\frac{e^2}{3} + \underline{43}$$

$$= \underline{41}\left(\underline{44}^2 f_1^2 + \underline{39}\,\underline{44}\,f_1 R_1 + \underline{39}^2\frac{R_1^2}{4}\right) + \underline{41}\,e\left(\underline{44}\,f_1 V + \underline{39}\,\frac{R_1}{2}V\right) + \underline{45}\,V^2 + \underline{42}\,N^2$$

$$= \underline{41}\,\underline{44}^2 f_1^2 + \underline{39}\,\underline{41}\,\underline{44}\,f_1 R_1 + \underline{39}^2\,\underline{41}\,\frac{1}{4}R_1^2 + e\,\underline{41}\,\underline{44}\,\frac{d}{2a}f_1^2 - \cancel{e\,\underline{41}\,\underline{44}\,\underline{35}\,\frac{d}{e}}f_1 R_1$$

$$+ e\,\underline{41}\,\underline{39}\,\frac{1}{4}\frac{d}{a}f_1 R_1 - \cancel{e\,\underline{41}\,\underline{39}\,\frac{1}{2}\,\underline{35}\,\frac{d}{e}}R_1^2 + \underline{45}\,\frac{d^2}{4a^2}f_1^2 - \cancel{\underline{45}\,\frac{d}{e}\,\underline{35}\,\frac{d}{e}}f_1 R_1 + \underline{45}\,\underline{35}^2\frac{d^2}{e^2}R_1^2 + \underline{42}\,\frac{f_1^2}{4}$$

$$= f_1^2\left(\underline{46} + \underline{49} + \underline{53} + \underline{56}\right) + f_1 R_1\left(\underline{47} - \underline{50} + \underline{51} - \underline{54}\right) + R_1^2\left(\underline{48} - \underline{52} + \underline{55}\right).$$

The strain energy of the spar in the other case was $\underline{6}f_8^2 + \underline{2}f_8f_9 + \underline{5}f_9^2$.
Then the difference in strain energy of the two cases is

$$U_1 - U_2 = (\underline{6} - \underline{36})f_8^2 + (\underline{2} - \underline{37})f_8f_9 + (\underline{5} - \underline{38})f_9^2 - \underline{57}f_1^2 - \underline{58}f_1R_1 - \underline{59}R_1^2.$$

$$U_2 = (\underline{1}^{60} + \underline{59})R_1^2 + (\underline{41}^{61} + \underline{57})f_1^2 + \underline{432}f_6^2 + (\underline{433}^{62} + \underline{36})f_8^2 + \underline{38}f_9^2 - \underline{421}f_1f_6 - \underline{420}f_1f_8 + \underline{431}f_6f_8 + \underline{37}f_8f_9 + \underline{58}f_1R_1$$

$$\frac{\partial U}{\partial X} = 2\underline{60}R_1\frac{\partial R_1}{\partial X} + 2\underline{61}f_1\frac{\partial f_1}{\partial X} + 8\underline{32}f_6\frac{\partial f_6}{\partial X} + 2\underline{62}f_8\frac{\partial f_8}{\partial X} + 2\underline{38}f_9\frac{\partial f_9}{\partial X} - 4\underline{21}f_1\frac{\partial f_6}{\partial X} - 4\underline{21}f_6\frac{\partial f_1}{\partial X} - 4\underline{20}f_1\frac{\partial f_8}{\partial X} - 4\underline{20}f_8\frac{\partial f_1}{\partial X} + 4\underline{31}f_6\frac{\partial f_8}{\partial X} + 4\underline{31}f_8\frac{\partial f_6}{\partial X} + \underline{37}f_8\frac{\partial f_9}{\partial X} + \underline{37}f_9\frac{\partial f_8}{\partial X} + \underline{58}f_1\frac{\partial R_1}{\partial X} + \underline{58}R_1\frac{\partial f_1}{\partial X}.$$

$$\frac{\partial^2 U}{\partial X\partial Y} = 2\underline{60}\frac{\partial R_1}{\partial Y}\frac{\partial R_1}{\partial X} + 2\underline{61}\frac{\partial f_1}{\partial Y}\frac{\partial f_1}{\partial X} + 8\underline{32}\frac{\partial f_6}{\partial Y}\frac{\partial f_6}{\partial X} + 2\underline{62}\frac{\partial f_8}{\partial Y}\frac{\partial f_8}{\partial X} + 2\underline{38}\frac{\partial f_9}{\partial Y}\frac{\partial f_9}{\partial X} - 4\underline{21}\frac{\partial f_1}{\partial Y}\frac{\partial f_6}{\partial X} - 4\underline{21}\frac{\partial f_6}{\partial Y}\frac{\partial f_1}{\partial X} - 4\underline{20}\frac{\partial f_1}{\partial Y}\frac{\partial f_8}{\partial X} - 4\underline{20}\frac{\partial f_8}{\partial Y}\frac{\partial f_1}{\partial X} + 4\underline{31}\frac{\partial f_6}{\partial Y}\frac{\partial f_8}{\partial X} + 4\underline{31}\frac{\partial f_8}{\partial Y}\frac{\partial f_6}{\partial X} + \underline{37}\frac{\partial f_8}{\partial Y}\frac{\partial f_9}{\partial X} + \underline{37}\frac{\partial f_9}{\partial Y}\frac{\partial f_8}{\partial X} + \underline{58}\frac{\partial f_1}{\partial Y}\frac{\partial R_1}{\partial X} + \underline{58}\frac{\partial R_1}{\partial Y}\frac{\partial f_1}{\partial X}.$$

$$\frac{\partial^2 U}{\partial F_1^2} = \underline{61} + .25\,\underline{62} + .25\,\underline{38}\frac{d^2}{a^2} + \underline{20} + \underline{20} - .25\,\underline{37}\frac{d}{a}$$

$$\frac{\partial^2 U}{\partial R_1\partial F_1} = -.353553\,\underline{62} + .353553\,\underline{38}\frac{d}{a} - 1.414214\,\underline{21} - 1.414214\,\underline{20} - .707107\,\underline{31} - \frac{.353553}{2}\underline{37} + \frac{.353553}{2}\underline{37}\frac{d}{a} + .707107\,\underline{58}.$$

$$\frac{\partial^2 U}{\partial R_1^2} = 2\underline{60} + 2\underline{32} + .5\,\underline{62} + .5\,\underline{38} + \underline{31} + \underline{31} + .25\,\underline{37} + .25\,\underline{37}.$$

$\underline{35} = \frac{54-8}{54} = .851851$; $\underline{35}^2 = .725651$; $\underline{35}^3 = .618148$

$\underline{36} = .851851 \times 28.4763\,g = 24.2576\,g$

$\underline{37} = .725651 \times 276.722\,g = 200.804\,g$

$\underline{38} = .618148\,(1681.767 - 125.187)\,g + .851851 \times 125.1865\,g = 962.1968\,g + 106.6402\,g = \underline{1068.837}\,g$

$\underline{39} = 29 - 6 = \underline{46}.$

$\underline{40} = \frac{8}{54} = .148148$

$\underline{41} = .148148 \times 1.60142\,g = .37725\,g$

$\underline{42} = .148148 \times 12.0778\ q = \underline{1.78930\ q}$

$\underline{43} = .148148 \times 125.1865\ q = \underline{18.5461\ q}$

$\underline{44} = \frac{3.2}{2}\left(\frac{46}{27} - 1\right) = \underline{1.12593}$

$\underline{45} = .23725\ q \times \frac{64}{3} + 18.5461\ q = 5.0613\ q + 18.5461\ q = \underline{23.6064\ q}$

$\underline{46} = .23725\ q \times 1.12593^2 = \underline{30.0766\ q}$

$\underline{47} = 46 \times .23725\ q \times 1.12593 = \underline{12.2878\ q}$

$\underline{48} = 46^2 \times .23725\ q \times .25 = \underline{125.5053\ q}$

$\underline{49} = 8 \times .23725\ q \times 1.12593 \times \frac{3.2}{54} = \underline{.126638\ q}$

$\underline{50} = .23725\ q \times 1.12593 \times .851851 \times 27 = \underline{6.1439\ q}$

$\underline{51} = 8 \times .23725\ q \times 46 \times .25 \times \frac{3.2}{27} = \underline{2.5869\ q}$

$\underline{52} = .23725\ q \times 46 \times .5 \times .851851 \times 27 = \underline{125.5052\ q}$

$\underline{53} = 23.6064\ q \times \left(\frac{3.2}{54}\right)^2 = \underline{.0828976\ q}$

$\underline{54} = 23.6064\ q \times 3.2 \times .851851 \times .125 = \underline{8.04352\ q}$

$\underline{55} = 23.6064\ q \times .725651 \times \frac{729}{64} = \underline{195.1211\ q}$

$\underline{56} = 1.78930 \times .25 = \underline{.447325\ q}$

$\underline{57} = (30.0766 + .1266 + .0829 + .4473)\ q = \underline{30.7334\ q}$

$\underline{58} = (12.2878 - 6.1439 + 2.5869 - 8.04352)\ q = \underline{.6873\ q}$

$\underline{59} = (125.5053 - 125.5052 + 195.1211)\ q = \underline{195.1211\ q}$

$\underline{60} = 1095.61\ q + 195.12\ q = \underline{1290.73\ q}$

$\underline{61} = 4 \times 1111.712 + 30.7334 = \underline{4477.581\ q}$

$\underline{62} = 4 \times 2968.5033 + 24.2576 = \underline{11,898.27\ q}$

$\frac{\partial^2 U}{\partial P_1^2} = q(4477.581 + 2974.568 + 3.753 + 3316.622) = \underline{.001107725}$

$\frac{\partial^2 U}{\partial P_1 R_1} = q(.353553)(-11,898.27 + 126.677 - 40,013.955 - 16,014.32 - 6633.24 - 5507.67 - 100.402 + 11.90 + 1.31) = \underline{-.0014144705}$

$\frac{\partial^2 U}{\partial R_1^2} = q(2581.46 + 7361.635 + 5949.135 + 534.419 + 5507.668 + 100.402) = \underline{.0022034472}$

$\delta_1 = 2P_1\left[.001107725 + (-.0014144705)\left(\frac{-.000168961}{-.000988289}\right) - \left(\frac{.642034}{.0022034472}\right)\right] = .000337922\ P_1$

$\frac{P_1}{\delta_1} = \underline{2959.\ \#/IN}$ (Does make a difference. @ 2¼ %).

The j^{th} cofactor is identical to the column of coefficients of of the j^{th} R in the expression for $\frac{\partial U}{\partial P}$ at the top of page "b." Observing that the coefficients of the P's are mirrored about a diagonal through the right-hand portions of $\left[\frac{\partial U}{\partial R_i}\right]_0$, it must follow that $\underline{\underline{i}}_j R_j$ of the first line can be replaced by $\frac{\partial R_j}{\partial P_i}\left(\underline{\underline{1}}_j P_1 + \underline{\underline{2}}_j P_2 + \cdots\cdots + \underline{\underline{n}}_j P_n\right)$.

Accordingly it may be seen that the expression for $\frac{\partial U}{\partial P_i}$ on page "c" is equivalent to the expression for $\delta_{i,\infty}$ at the bottom of page "a". This peculiarity offers a desirable way of checking the results of a particular computation; e.g.,

$$f_1 = \frac{\partial \delta_i}{\partial P_j} - \frac{\partial^2 U}{\partial P_i \partial P_j} = \frac{\partial^2 U}{\partial P_i \partial R_1}\frac{\partial R_1}{\partial P_j} + \cdots\cdots + \frac{\partial^2 U}{\partial R_i \partial R_m}\frac{\partial R_m}{\partial P_j} = \frac{\partial^2 U}{\partial P_j \partial R_1}\frac{\partial R_1}{\partial P_i} + \cdots\cdots + \frac{\partial^2 U}{\partial P_j \partial R_m}\frac{\partial R_m}{\partial P_i}.$$

Rewriting this equality using the notation introduced on page 13,

$$\underline{U_{P_1R_1}}\,\underline{R_1P_j} + \cdots\cdots + \underline{U_{P_iR_m}}\,\underline{R_mP_j} = \underline{U_{P_jR_1}}\,\underline{R_1P_i} + \cdots\cdots + \underline{U_{P_jR_m}}\,\underline{R_mP_i}$$

Example: Let $i = 2$; and $j = 4$;
Using results on pages 24 to 30,

$$\underline{U_{P_2R_1}}\,\underline{R_1Q_2} + \underline{U_{P_2R_2}}\,\underline{R_2Q_2} + \underline{U_{P_2R_3}}\,\underline{R_3Q_2} = \underline{U_{Q_2R_1}}\,\underline{R_1P_2} + \underline{U_{Q_2R_2}}\,\underline{R_2P_2} + \underline{U_{Q_2R_3}}\,\underline{R_3P_2}$$

$$\begin{array}{ll} (-50,211.14)_{\mathcal{G}}(.00076635-1) & (-16.85307)_{\mathcal{G}}(.642773) \\ (\ 16,503.03)_{\mathcal{G}}(.000000000) & (\ 0.0\ )_{\mathcal{G}}(.051070) \\ (\ 72,152.15)_{\mathcal{G}}(.00038317\,6) & (\ 0.0\ )_{\mathcal{G}}(-.054695) \end{array}$$

$$-38.47936 + 0 + 27.64697 = -10.83270 + 0 + 0$$
$$-10.8324 = -10.8327 \quad \text{OK.}$$

$\theta_1 = 180 - 2\sin^{-1}\frac{a_2}{\sqrt{a_2^2 + a_3^2}} =$

$r_1 = \frac{a_2}{\sin\theta_1} =$

$\theta_3 = 180 - 2\sin^{-1}\frac{a_5}{\sqrt{a_5^2 + a_4^2}} =$

$r_2 = \frac{a_5}{\sin\theta_3} =$

$\theta_2 = \theta_3 + \sin^{-1}\frac{a_4}{r_2} =$

$\beta = \tan^{-1}\frac{r_2 + a_2 - a_4}{r_1 + a_5 - a_3} =$

$\underline{3} = a_{25} - a_{20} =$

$\underline{4} = \frac{1}{2\cos\delta} =$

$\underline{5} = \frac{1}{2\sin\delta} =$

$\underline{6} = \cos\beta =$

$\underline{7} = \sin\beta =$

$\underline{8} = a_{20} + a_{17} + a_2 + a_{13} =$

$\underline{9} = a_3 - a_{14} + a_{15} =$

$\underline{10} = \underline{11} - \underline{7} =$

$\underline{11} = \frac{\underline{6}\,\underline{8} + \underline{7}\,\underline{9}}{2a_{16}} =$

$\underline{12} = \frac{a_{25}}{2a_{16}} =$

$\underline{13} = \frac{1}{2a_{16}} =$

$\underline{14} = a_2 + a_{13} =$

$\underline{15} = 2a_{16} - \underline{9} =$

$\underline{16} = \underline{9} =$

$\underline{17} = a_{25} - a_{20} - a_{17} =$

$\underline{18} = \frac{\underline{7}\,\underline{15} - \underline{6}\,\underline{14}}{2a_{16}} =$

$\underline{19} = -\frac{\underline{6}\,\underline{14} + \underline{7}\,\underline{16}}{2a_{16}} =$

$\underline{20} = \frac{\underline{17}}{2a_{16}} =$

$\underline{21} = \underline{11} =$

$\underline{22} = \underline{10} =$

$\underline{23} = \underline{19} + \underline{7} =$

$\underline{24} = \underline{18} - \underline{7} =$

$\underline{25} = \dfrac{L}{2 A_{ST} E_{ST}} =$

$\underline{26} = 2\,\underline{25}\,\underline{5}^2 =$

$\underline{27} = 2\,\underline{25}\,\underline{4}^2 =$

$\underline{28} = \tan\alpha =$

$\underline{29} = \sin\alpha =$

$\underline{30} = \cos\alpha =$

$\underline{31} = \dfrac{a_{27}}{\underline{30}} + a_{20} =$

$\underline{32} = \dfrac{1}{2\,\underline{30}\,E_{SP}\,I_{SP}} =$

$\underline{33} = \dfrac{a_{16}}{2\,\underline{30}\,A_{SP}\,E_{SP}} =$

$\underline{34} = \dfrac{a_{16}\alpha_{SP}}{2\,\underline{30}\,A_{SP}\,G_{SP}} =$

$\underline{35} = \underline{32}\,\underline{31}^2 a_{16} =$

$\underline{36} = \underline{32}\,\underline{31}\,a_{16}^2 =$

$\underline{37} = \underline{36}\,\underline{28} =$

$\underline{38} = \underline{32}\,\dfrac{a_{16}^3}{3} =$

$\underline{39} = \underline{38}\,\underline{28}^2 =$

$\underline{40} = \underline{33}\,\underline{30}^2 =$

$\underline{299} = 2\,\underline{38}\,\underline{28} =$

$\underline{41} = 2\,\underline{33}\,\underline{30}\,\underline{29} =$

$\underline{42} = \underline{33}\,\underline{29}^2 =$

$\underline{43} = \underline{34}\,\underline{29}^2 =$

$\underline{44} = 2\,\underline{34}\,\underline{29}\,\underline{30} =$

$\underline{45} = \underline{34}\,\underline{30}^2 =$

$\underline{46} = \underline{35} + \underline{37} + \underline{39} + \underline{40} + \underline{43} =$

$\underline{47} = \underline{36} + \underline{299} - \underline{41} + \underline{44} =$

$\underline{48} = \underline{38} + \underline{42} + \underline{45} =$

$\underline{49} = \underline{10} + \underline{18} =$

$\underline{50} = \underline{11} + \underline{19} =$

$\underline{51} = \underline{20} - \underline{12} =$

$\underline{52} = \underline{21} + \underline{24} =$

$\underline{53} = \underline{22} + \underline{23} =$

$\underline{55} = \tan \Omega = \dfrac{a_{14} - a_{15}}{a_{13}} =$

$\underline{56} = \sin \Omega =$

$\underline{57} = \cos \Omega =$

$\underline{82} = \dfrac{a_{13}^3}{6 E_{FO} I_{FO_1} \underline{57}} =$

$\underline{83} = \dfrac{a_{13}}{2 A_{FO_1} E_{FO} \underline{57}} =$

$\underline{84} = \dfrac{a_{13} \alpha_{FO_1}}{2 A_{FO_1} G_{FO} \underline{57}} =$

$\underline{85} = \underline{82} + \underline{83}\,\underline{56}^2 + \underline{84}\,\underline{57}^2 =$

$\underline{58} = 2\,\underline{55}\,\underline{82} + 2\,\underline{56}\,\underline{57}(\underline{84} - \underline{83}) =$

$\underline{59} = \underline{82}\,\underline{55}^2 + \underline{83}\,\underline{57}^2 + \underline{84}\,\underline{56}^2 =$

$\underline{86} = r_1 + a_{15} - a_{14} =$

$\underline{87} = \dfrac{r_1}{2 E_{FO} I_{FO_1}} =$

$\underline{88} = \dfrac{r_1}{2 A_{FO_1} E_{FO}} =$

$\underline{89} = \dfrac{r_1 \alpha_{FO_1}}{2 A_{FO_1} G_{FO}} =$

$\underline{90} = .5(\beta_1 - .5 \sin 2\beta_1) =$

$\underline{91} = .5(\beta_1 + .5 \sin 2\beta_1) =$

$\underline{92} = 1 - \cos \beta_1 =$

$\underline{93} = \sin \beta_1 =$

$\underline{94} = .5(1 - \cos 2\beta_1) =$

$\underline{95} = \underline{87}\,\beta_1\,a_{13}^2 =$

$\underline{96} = 2\,\underline{87}\,\beta_1\,a_{13}\,\underline{86} =$

$\underline{97} = \underline{87}\,\beta_1\,\underline{86}^2 =$

$\underline{98} = \underline{87}\,\underline{90}\,r_1^2 =$

$\underline{99} = \underline{87}\,\underline{91}\,r_1^2 =$

$\underline{100} = 2\,\underline{87}\,\underline{92}\,r_1\,a_{13} =$

$\underline{101} = 2\,\underline{87}\,\underline{92}\,r_1\,\underline{86} =$

$\underline{102} = 2\,\underline{87}\,\underline{93}\,r_1\,a_{13} =$

$\underline{103} = 2\,\underline{87}\,\underline{93}\,r_1\,\underline{86} =$

104 = 87 94 r_1^2 =
105 = 88 90 =
106 = 88 91 =
107 = 88 94 =
108 = 89 90 =
109 = 89 91 =
110 = 89 94 =

111 = 95 + 98 + 100 + 105 + 109 =
112 = −96 − 101 + 102 + 104 + 107 − 110 =
113 = 97 + 99 − 103 + 106 + 108 =

114 = $\cos B_3$ =
115 = $\sin B_3$ =

116 = $\dfrac{r_2}{2 E_{FO} I_{FO_2}}$ =

117 = $\dfrac{r_2}{2 A_{FO_2} E_{FO}}$ =

118 = $\dfrac{r_2 \alpha_{FO_2}}{2 A_{FO_2} G_{FO}}$ =

119 = $.5(B_2 - .5 \sin 2B_2)$ =
120 = $.5(B_2 + .5 \sin 2B_2)$ =
121 = $1 - \cos B_2$ =
122 = $\sin B_2$ =
123 = $.5(1 - \cos 2B_2)$ =
124 = 116 B_2 =
125 = 2 116 B_2 r_2 =
126 = 116 B_2 r_2^2 =
127 = 116 119 r_2^2 =
128 = 116 120 r_2^2 =
129 = 2 116 121 r_2 =
130 = 2 116 121 r_2^2 =
131 = 2 116 122 r_2 =
132 = 2 116 122 r_2^2 =
133 = 116 123 r_2^2 =
134 = 117 119 =

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135 = 117 120 =

136 = 117 123 =

137 = 118 119 =

138 = 118 120 =

139 = 118 123 =

140 = 126 + 128 − 132 + 135 + 137 =

141 = 127 + 134 + 138 =

142 = −125 + 131 =

143 = 130 − 133 − 136 + 139 =

144 = 124 14^2 =

145 = 124 9^2 =

146 = 2 124 9 14 =

147 = 140 115^2 =

148 = 141 114^2 =

149 = 143 114 115 =

150 = 147 + 148 + 149 =

151 = 140 114^2 =

152 = 141 115^2 =

153 = 143 114 115 =

154 = 151 + 152 − 153 =

155 = 2 140 114 115 =

156 = 2 141 114 115 =

157 = 143 $(114^2 - 115^2)$ =

158 = 155 − 156 + 157 =

159 = 142 14 115 =

160 = 129 14 114 =

161 = 159 − 160 =

162 = 142 14 114 =

163 = 129 14 115 =

164 = 162 + 163 =

165 = 142 9 115 =

166 = 129 9 114 =

167 = −165 + 166 =

168 = 142 9 114 =

169 = 129 9 115 =

170 = −168 − 169 =

171 = 144 + 154 + 164 =

FORM 1194

172 = 145 + 150 + 167 =

173 = 7 ² 150 + 6 ² 154 − 6 7 158 =

174 = −146 + 158 + 161 + 170 =

175 = 2 6 154 − 7 (158 + 161) + 6 164 =

176 = 6 (158 + 170) − 7 (2 150 + 167) =

177 = $a_{16} - a_7$ =

178 = 14 + a_{29} =

179 = $a_5 + a_6$ =

180 = 6 a_{29} + 7 179 =

181 = $\dfrac{1}{2 E_{FO} I_{FO_3}}$ =

182 = $\dfrac{a_7}{2 A_{FO_3} E_{FO}}$ =

183 = $\dfrac{a_7 \alpha_{FO_3}}{2 A_{FO_3} G_{FO}}$ =

184 = 181 a_7 =

185 = 184 a_7 =

186 = 185 $\dfrac{a_7}{3}$ + 183 =

187 = 184 178 ² =

188 = 184 180 ² =

189 = 184 177 ² =

190 = 2 184 178 180 =

191 = 2 184 178 177 =

192 = 2 184 180 177 =

193 = 185 178 =

194 = 185 180 =

195 = 185 177 =

196 = 185 7 178 =

197 = 185 7 180 =

198 = 185 7 177 =

199 = 2 186 7 =

200 = 186 7 ² =

201 = 2 182 6 =

202 = 182 6 ² =

203 = 187 + 182 =

204 = 188 + 197 + 200 + 202 =

205 = 189 + 195 + 186 =
206 = 190 + 196 + 201 =
207 = −191 − 193 =
208 = −192 − 194 − 198 − 199 =
209 = 85 + 111 + 171 + 203 =
210 = 173 + 204 =
211 = 59 + 113 + 172 + 205 =
212 = 175 + 206 =
213 = 58 + 112 + 174 + 207 =
214 = 176 + 208 =
300 = 209 + 46 =
301 = 2 210 =
302 = 2 26 =
303 = 2 27 =
304 = 2 300 =
305 = 2 211 =
306 = 2 48 =

UP_1^2 = + 301 =
+ 2 18 214 =
+ 305 $(18^2 + 24^2)$ =
− 2 6 24 213 =
+ 6^2 304 =
− 2 6 47 52 =
+ 306 $(49^2 + 52^2)$ =

UPP_2 = + 214 (19 + 24) =
+ 305 (18 19 + 23 24) =
− 6 213 (23 + 24) =
+ 6^2 304 =
− 6 212 =
− 6 47 (52 + 53) =
+ 306 (49 50 + 52 53) =

UP_1Q_1 = + 20 214 =
+ 20 305 (18 + 24) =
+ 213 (24 − 6 20) =
− 6 304 =
− 47 (6 51 − 52) =
+ 306 51 (49 + 52) =

UP_1Q_2 = + 6 13 47 =
− 13 306 (49 + 52) =

UP_1R_1 = 20 214 =
+ 20 305 (18 + 24) =
+ 213 (24 − 6 20) =
− 6 304 =
− 47 (6 20 − 52) =
+ 20 306 (49 + 52) =

UP_1R_2 = − .5 214 =
− .5 305 (18 − 24) =
− .5 6 213 =
− .5 6 47 =
− .5 306 (49 + 52) =

UP_1R_3 = + 212 =
+ 213 (18 − 24) =
+ 6 304 =
+ 47 (49 − 52) =

UP_3^2 = + 301 =
+ 2 23 214 =
− 2 6 212 =
+ 305 $(19^2 + 23^2)$ =
− 2 6 23 213 =
+ 6^2 304 =
− 2 6 47 53 =
+ 306 $(50^2 + 53^2)$ =

UP_2Q_1 = + 20 214 =
+ 212 =
+ 20 305 (19 + 23) =
+ 213 (23 − 6 20) =
− 6 304 =
− 47 (6 51 − 53) =
+ 51 306 (50 + 53) =

UP_2Q_2 = + 6 13 47 =
− 13 306 (50 + 53) =

UP_2R_1 = + 20 214 =
+ 212 =
+ 20 305 (19 + 23) =
+ 213 (23 − 6 20) =
− 6 304 =
− 47 (6 20 − 53) =
+ 20 306 (50 + 53) =

UP_2R_2 = + .5 214 =
− .5 305 (19 − 23) =
− .5 6 213 =
− .5 6 47 =
− .5 306 (50 − 53) =

UP_2R_3 = -212 =
$+213(19-23)$ =
$+6\,304$ =
$+47(50-53)$ =

UQ_1^2 = $2\,20^2\,305$ =
$+2\,20\,213$ =
$+304$ =
$+2\,47\,51$ =
$+2\,51^2\,306$ =

UQ_1Q_2 = $-13\,47$ =
$-2\,13\,51\,306$ =

UQ_1R_1 = $2\,20^2\,305$ =
$+2\,20\,213$ =
$+304$ =
$+47(20+51)$ =
$+2\,20\,51\,306$ =

UQ_1R_2 = $.5\,213$ =
$+.5\,47$ =

UQ_1R_3 = -304 =

UQ_2^2 = $2\,13^2\,306$ =

UQ_2R_1 = $-2\,13\,20\,306$ =
$-13\,47$ =

$UQ_2R_2 = 0 = UQ_2R_3$

UR_1^2 = + 302 =
+ 2 20² 305 =
+ 2 30 213 =
+ 304 =
+ 2 30 47 =
+ 2 20² 306 =

UR_1R_2 = + .5(213 + 47) =

UR_1R_3 = − 304 =

UR_2^2 = + 303 =
+ .5(305 + 306) =

UR_2R_3 = − 213 − 47 =

UR_3^2 = 2 304 =

$$\left.\begin{aligned} 0 &= u_1 + UR_1^2 R_1 + UR_1R_2 R_2 + UR_1R_3 R_3 \\ 0 &= u_2 + UR_1R_2 R_1 + UR_2^2 R_2 + UR_2R_3 R_3 \\ 0 &= u_3 + UR_1R_3 R_1 + UR_2R_3 R_2 + UR_3^2 R_3 \end{aligned}\right\} \quad u_i = UR_iP_iP_i + UR_iP_2P_2 + UR_iQ_1Q_1 + UR_iQ_2Q_2 .$$

$$\delta_1 = \underline{UP_1^2}\,P_1 + \underline{UP_1P_2}\,P_2 + \underline{UP_1Q_1}\,Q_1 + \underline{UP_1Q_2}\,Q_2 + \underline{UP_1R_1}\,R_1 + \underline{UP_1R_2}\,R_2 + \underline{UP_1R_3}\,R_3 .$$

$$\delta_2 = \underline{UP_1P_2}\,P_1 + \underline{UP_2^2}\,P_2 + \underline{UP_2Q_1}\,Q_1 + \underline{UP_2Q_2}\,Q_2 + \underline{UP_2R_1}\,R_1 + \underline{UP_2R_2}\,R_2 + \underline{UP_2R_3}\,R_3 .$$

$$\delta_3 = \underline{UP_1Q_1}\,P_1 + \underline{UP_2Q_1}\,P_2 + \underline{UQ_1^2}\,Q_1 + \underline{UQ_1Q_2}\,Q_2 + \underline{UQ_1R_1}\,R_1 + \underline{UQ_1R_2}\,R_2 + \underline{UQ_1R_3}\,R_3 .$$

$$\delta_4 = \underline{UP_1Q_2}\,P_1 + \underline{UP_2Q_2}\,P_2 + \underline{UQ_1Q_2}\,Q_1 + \underline{UQ_2^2}\,Q_2 + \underline{UQ_2R_1}\,R_1 + \underline{UQ_2R_2}\,R_2 + \underline{UQ_2R_3}\,R_3 .$$

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Station 742.5

Spar Properties:

Drawing No 7-0162-156. Referring to report # 7-0562-4, "Distribution of the Fin Roll Moment",

$$\int_0^{\frac{L}{2}} \frac{x^2}{I_y}\,dx = 6560\ in^{-1} = \frac{1}{I_c}\int_0^{\frac{L}{2}} x^2\,dx = \frac{1}{I_c}\,\frac{L^3}{24}$$

$$\therefore I_c = \frac{114^3}{24 \times 6560} = 9.42\ in^4 \quad \text{(Equivalent spar of constant "I")}$$

Say section 5, sheet 32 of ref. report for spar properties; i.e., $A = 2.1855$; $I = 9.3179$;

$$\alpha_{sp} = \frac{A}{It}A'\bar{y}' = \frac{2.1855}{9.3179 \times .125}\left(\overset{1.643}{.755 \times 2.1765} + \overset{.629}{.125 \times 1.989} + \overset{2.535}{.125 \times 2.0515 \times 1.025}\right) = 4.75$$

Former Properties:

Dwg. 7-0158-133

$A = 4 \times 1.25 + 1.6 \times .1 = .5 + .16 = .66\ in^2$.

$$I = \frac{1}{12}\left(2.5 \times 4 - 1.6^3 \times 1.15\right) = \frac{1}{12}\left(10 - 4.716\right) = .440\ in^4$$

$$\alpha_{FO} = \frac{.66}{.44 \times .1}\left(\overset{.225}{.9 \times .2 \times 1.25} + \overset{.032}{.4 \times .1 \times .8}\right) = 3.855.$$

These properties for FO_0, FO_1, and FO_2.
Change 2" width to 4" for FO_3.

$$A_{FO_3} = .66 + .1 \times 2 = .86$$

$$I_{FO_3} = \frac{1}{12}\left(1.25 \times 6.4 - 1.15 \times 3.6^3\right) = \frac{1}{12}\left(80 - 53.7\right) = 2.19$$

$$\alpha_{FO_3} = \frac{.86}{2.19 \times .1}\left(\overset{.475}{1.9 \times .2 \times 1.25} + \overset{.120}{0.95 \times .1 \times 1.9}\right) = 1.16$$

Strut Properties:

$L = 42.5^\circ$, $\gamma = 9.2^\circ$

$A_{ST} = .3136\ in^2$. (O.D. = 1.75 ; .058 wall); $E_{ST} = 29 \times 10^6$ p.s.i.

Geometry:

$a_2 = 21.8$	$a_7 = 11.32$	$a_{17} = 7.24$	$a_{29} = -2.1"$
$a_3 = 8.8$	$a_{11} = 5.4$	$a_{20} = 2.4$	
$a_4 = 7.5$	$a_{13} = 23"$	$a_{25} = 54.4$	
$a_5 = 17.4$	$a_{15} - a_{14} = -.32$	$a_{26} = 47.9"$	
$a_6 = 19.2$	$a_{16} = 56.4$	$a_{27} = .9$	

FORM 17104

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$\theta_1 = 180 - 2\sin^{-1}\frac{a_2}{\sqrt{a_2^2+a_3^2}} = 180 - 2\sin^{-1}\frac{21.8}{\sqrt{475.24+77.44}} = 180 - 2\sin^{-1}.92730 = 43°58' = .76736$ rad. $(2\times 68°1')$

$r_1 = \frac{a_2}{\sin\theta_1} = \frac{21.8}{.69424} = 31.401''$

$\theta_3 = 180 - 2\sin^{-1}\frac{a_5}{\sqrt{a_6^2+a_4^2}} = 180 - 2\sin^{-1}\frac{17.4}{\sqrt{302.76+56.25}} = 180 - 2\sin^{-1}.91830 = 46°38' = .81390$ rad. $(2\times 66°41')$

$r_2 = \frac{a_5}{\sin\theta_3} = \frac{17.4}{.72697} = 23.935''$

$\theta_2 = \theta_3 + \sin^{-1}\frac{a_6}{r_2} = 46°38' + \sin^{-1}\frac{19.2}{23.935} = 46°38' + \sin^{-1}.80217 = 99°58'3' = 1.74483$ rad. $(53°20.3')$

$\beta = \tan^{-1}\frac{r_2+a_3-a_4}{r_1+a_5-a_3} = \tan^{-1}\frac{23.935+21.8-7.5}{31.401+17.4-8.8} = \tan^{-1}.95585 = 43°42.4'$ (38.235 / 40.001)

$\underline{3} = a_{25} - a_{20} = 54.4 - 2.4 = 52.0$

$\underline{4} = \frac{1}{2\cos\delta} = \frac{1}{2\cos 9.2°} = \frac{1}{2\times.98714} = .506573$

$\underline{5} = \frac{1}{2\sin\delta} = \frac{1}{2\sin 9.2°} = \frac{1}{2\times.15958} = 3.127345$

$\underline{6} = \cos\beta = \cos 43°42.4' = .72289$

$\underline{7} = \sin\beta = \sin 43°42.4' = .69096$

$\underline{8} = a_{20} + a_{17} + a_2 + a_{13} = 2.4 + 7.24 + 21.8 + 23 = 54.44$

$\underline{9} = a_3 - a_{14} + a_{15} = 8.8 - .32 = 8.48$

$\underline{10} = \underline{11} - \underline{7} = .40083 - .69096 = -.29013$

$\underline{11} = \frac{\underline{6}\,\underline{8} + \underline{7}\,\underline{9}}{2a_{16}} = \frac{.72289\times 54.44 + .69096\times 8.48}{2\times 56.4} = .40083$

$\underline{12} = \frac{a_{25}}{2a_{16}} = \frac{54.4}{112.8} = .48227$

$\underline{13} = \frac{1}{2a_{16}} = \frac{1}{112.8} = .0088624$

$\underline{14} = a_2 + a_{13} = 21.8 + 23 = 44.8$

$\underline{15} = 2a_{16} - \underline{9} = 112.8 - 8.48 = 104.32$

$\underline{16} = \underline{9} = 8.48$

$\underline{17} = a_{25} - a_{20} - a_{17} = 54.4 - 2.4 - 7.24 = 44.76$

$\underline{18} = \frac{\underline{7}\,\underline{15} - \underline{6}\,\underline{14}}{2a_{16}} = \frac{.69096\times 104.32 - .72289\times 44.8}{112.8} = .351910$

$\underline{19} = -\frac{\underline{6}\,\underline{14} + \underline{7}\,\underline{16}}{2a_{16}} = -\frac{.72289\times 44.8 + .69096\times 8.48}{112.8} = -.339050$

$\underline{20} = \frac{\underline{17}}{2a_{16}} = \frac{44.76}{112.8} = .3968085$

$\underline{21} = \underline{11} = .40083$

$\underline{22} = \underline{10} = -.29013$

$\underline{23} = \underline{19} + \underline{7} = -.33905 + .69096 = .35191$

$\underline{24} = \underline{18} - \underline{7} = .35191 - .69096 = -.33905$

$\underline{25} = \dfrac{L}{2 A_{ST} E_{ST}} = \dfrac{42.5}{2 \times .3136 \times 2.9 \times 10^7} = 23.366\ q$. $(q = 10^{-7})$

$\underline{26} = 2\,\underline{25}\,\underline{5}^2 = 2 \times 23.366\ q\ 3.127345^2 = 457.052\ q$.

$\underline{27} = 2\,\underline{25}\,\underline{4}^2 = 2 \times 23.366\ q\ .506513^2 = 11.8493\ q$.

$\underline{28} = \tan \alpha = \tan 4^\circ = .06993$

$\underline{29} = \sin \alpha = \sin 4^\circ = .06976$

$\underline{30} = \cos \alpha = \cos 4^\circ = .99756$

$\underline{31} = \dfrac{a_{37}}{\underline{30}} + a_{20} = \dfrac{.90}{.99756} + 2.4 = 3.3022$

$\underline{32} = \dfrac{1}{2\,\underline{30}\,E_{SP}\,I_{SP}} = \dfrac{1}{2 \times .99756 \times 10^7 \times 9.3179} = .0537914\ q$.

$\underline{33} = \dfrac{a_{16}}{2\,\underline{30}\,A_{SP}\,E_{SP}} = \dfrac{56.4}{2 \times .99756 \times 2.1855 \times 10^7} = 12.9348\ q$.

$\underline{34} = \dfrac{a_{16} \alpha_{SP}}{2\,\underline{30}\,A_{SP}\,G_{SP}} = \dfrac{56.4 \times 4.75}{2 \times .99756 \times 2.1855 \times .4 \times 10^7} = 153.6008\ q$.

$\underline{36} = \underline{32}\,\underline{31}^2 a_{16} = .0537914\ q\ 3.3022^2 \times 56.4 = 33.0825\ q$

$\underline{36} = \underline{32}\,\underline{31}\,a_{16}^2 = .0537914\ q\ 3.3022 \times 56.4^2 = 565.0338\ q$

$\underline{37} = \underline{36}\,\underline{28} = 565.0338\ q \times .06993 = 39.5128\ q$

$\underline{38} = \underline{32}\,\dfrac{a_{16}^3}{3} = .0537914\ q\ .33333 \times 56.4^3 = 3216.8325\ q$

$\underline{39} = \underline{38}\,\underline{28}^2 = 3216.8325\ q\ .06993^2 = 15.73095\ q$

$\underline{40} = \underline{33}\,\underline{30}^2 = 12.9348\ q \times .99756^2 = 12.8718\ q$

$\underline{299} = 2\,\underline{38}\,\underline{28} = 2 \times 3216.8325\ q\ .06993 = 449.9062\ q$

$\underline{41} = 2\,\underline{33}\,\underline{30}\,\underline{29} = 2 \times 12.9348\ q\ .99756 \times .06976 = 1.8003\ q$

$\underline{42} = \underline{33}\,\underline{29}^2 = 12.9348\ q\ .06976^2 = .062947\ q$

$\underline{43} = \underline{34}\,\underline{29}^2 = 153.6008\ q\ .06976^2 = .747492\ q$

$\underline{44} = 2\,\underline{34}\,\underline{29}\,\underline{30} = 2 \times 153.6008\ q\ .06976 \times .99756 = 21.3781\ q$

$\underline{45} = \underline{34}\,\underline{30}^2 = 153.6008\ q\ .99756^2 = 152.8521\ q$

$\underline{46} = \underline{35} + \underline{37} + \underline{39} + \underline{40} + \underline{43} = q(33.0825 + 39.5128 + 15.73095 + 12.8718 + .7475) = 101.9456\ q$

$\underline{47} = \underline{36} + \underline{299} - \underline{41} + \underline{44} = q(565.0338 + 449.9062 - 1.8003 + 21.3781) = 1034.5178\ q$

$\underline{48} = \underline{38} + \underline{42} + \underline{45} = q(3216.8325 + .0629 + 152.8521) = 3369.7475\ q$

$\underline{49} = \underline{10} + \underline{18} = -.29013 + .35191 = +.06178$

$\underline{50} = \underline{11} + \underline{19} = .40083 - .33905 = +.06178$

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$\underline{57} = \underline{20} - \underline{12} = .3968085 - .48227 = -.08546$

$\underline{52} = \underline{21} + \underline{24} = .40083 - .33905 = +.06178$

$\underline{53} = \underline{22} + \underline{23} = -.29013 + .35191 = +.06178$

$\underline{55} = \tan \Omega = \frac{a_{14} - a_{15}}{a_{13}} = \frac{.32}{23} = .013913 \quad . \quad (\Omega = 0°48')$

$\underline{56} = \sin \Omega = .013913$

$\underline{57} = \cos \Omega = .99990$

$\underline{82} = \frac{a_{13}^3}{6 E_{FO} I_{FO_2} \underline{57}} = \frac{23^3}{6 \times 10^7 \times .440 \times .9999} = 4,609.16 \text{ q}$

$\underline{83} = \frac{a_{13}}{2 A_{FO_2} E_{FO} \underline{57}} = \frac{23}{2 \times .66 \times 10^7 \times .9999} = 17.4260 \text{ f}$

$\underline{84} = \frac{a_{13} \alpha_{FO_2}}{2 A_{FO_2} G_{FO} \underline{57}} = \frac{23 \times 3.855}{2 \times .66 \times .4 \times 10^7 \times .9999} = 167.9431 \text{ q}$

$\underline{85} = \underline{82} + \underline{83}\,\underline{56}^2 + \underline{84}\,\underline{57}^2 = q\,(4609.16 + 17.4260 \times .013913^2 + 167.9431 \times .9999^2) = 4,777.07 \text{ q}$

$\underline{58} = 2\,\underline{56}\,\underline{82} + 2\,\underline{56}\,\underline{57}\,(\underline{84} - \underline{83}) = q\,[2 \times .013913 \times 4609.16 + 2 \times .013913 \times .9999 (167.9431 - 17.4260)] = 132.4424 \text{ q}$

$\underline{59} = \underline{82}\,\underline{55}^2 + \underline{83}\,\underline{57}^2 + \underline{84}\,\underline{56}^2 = q\,(4609.16 \times .013913^2 + 17.4260 \times .9999^2 + 167.9431 \times .013913^2) = 18.3472 \text{ q}$

$\underline{86} = r_1 + a_{15} - a_{10} = 31.401 - .32 = 31.081$

$\underline{87} = \frac{r_1}{2 E_{FO} I_{FO_1}} = \frac{31.401}{2 \times 10^7 \times .44} = 35.6829 \text{ f}$

$\underline{88} = \frac{r_1}{2 A_{FO_1} E_{FO}} = \frac{31.401}{2 \times .66 \times 10^7} = 23.7886 \text{ f}$

$\underline{89} = \frac{r_1 \alpha_{FO_1}}{2 A_{FO_1} G_{FO}} = \frac{31.401 \times 3.855}{2 \times .66 \times .4 \times 10^7} = 229.2626 \text{ f}$

$\underline{90} = .5\,(B_1 - .5 \sin 2B_1) = .5\,(.76736 - .5 \times .99935) = .13384$

$\underline{91} = .5\,(B_1 + .5 \sin 2B_1) = .5\,(.76736 + .5 \times .99935) = .63352$

$\underline{92} = 1 - \cos B_1 = 1 - .71974 = .28026$

$\underline{93} = \sin B_1 = .69424$

$\underline{94} = .5\,(1 - \cos 2B_1) = .5\,(1 - .03606) = .48197$

$\underline{95} = \underline{87}\,B_1\,a_{13}^2 = 35.6829 \text{ f} \cdot .76736 \times 23^2 = 14,484.87 \text{ f}$

$\underline{96} = 2\,\underline{87}\,B_1\,a_{13}\,\underline{86} = 2 \times 35.6829 \text{ f} \cdot .76736 \times 23 \times 31.081 = 38,148.20 \text{ q}$

$\underline{97} = \underline{87}\,B_1\,\underline{86}^2 = 35.6829 \text{ f} \cdot .76736 \times 31.081^2 = 26,451.41 \text{ f}$

$\underline{98} = \underline{87}\,\underline{90}\,r_1^2 = 35.6829 \text{ f} \cdot .13384 \times 31.401^2 = 4,709.05 \text{ f}$

$\underline{99} = \underline{87}\,\underline{91}\,r_1^2 = 35.6829 \text{ f} \cdot .63352 \times 31.401^2 = 22,289.86 \text{ f}$

$\underline{100} = 2\,\underline{87}\,\underline{92}\,r_1\,a_{13} = 2 \times 35.6829 \text{ f} \cdot .28026 \times 31.401 \times 23 = 14,445.17 \text{ f}$

$\underline{101} = 2\,\underline{87}\,\underline{92}\,r_1\,\underline{86} = 2 \times 35.6829 \text{ f} \cdot .28026 \times 31.401 \times 31.081 = 19,520.44 \text{ f}$

$\underline{102} = 2\,\underline{87}\,\underline{93}\,r_1\,a_{13} = 2 \times 35.6829 \text{ f} \cdot .69424 \times 31.401 \times 23 = 35,782.53 \text{ f}$

$\underline{103} = 2\,\underline{87}\,\underline{93}\,r_1\,86 = 2 \times 35.6829 \text{ f} \cdot .69424 \times 31.401 \times 31.081 = 48,354.64 \text{ f}$

104 = 87 94 r_1^2 = 35,682.4 q .48197 × 31.401² = 16,957.70 q

105 = 88 90 = 23.7886 q .13384 = 3.18387 q

106 = 88 91 = 23.7886 q .63352 = 15.07055 q

107 = 88 94 = 23.7886 q .48197 = 11.46547 q

108 = 89 90 = 227.2626 q .13384 = 30.6845 q

109 = 89 91 = 229.2626 q .63352 = 145.2424 q

110 = 89 94 = 229.2626 q .48197 = 110.4977 q

111 = 95 + 98 + 100 + 105 + 109 = q(14,484.87 + 4,709.05 + 14,445.17 + 3.18 + 145.24) = 33,787.51 q

112 = −96 − 101 + 102 + 104 + 107 − 110 = q(−39,148.20 − 19,530.44 + 35,782.53 + 16,957.70 + 11.47 − 110.50) = −6,027.44 q

113 = 97 + 99 − 103 + 106 + 108 = q(26,451.41 + 22,289.86 − 48,354.64 + 15.07 + 30.68) = 432.38 q

114 = cos θ_3 = .68666

115 = sin θ_3 = .72697

$$116 = \frac{r_2}{2E_{FO}I_{FO_2}} = \frac{23.935}{2\times10^7\times.44} = 27.1988\ q$$

$$117 = \frac{r_2}{2A_{FO_2}E_{FO}} = \frac{23.935}{2\times.66\times10^7} = 18.1326\ q$$

$$118 = \frac{r_3 q_{FO_3}}{2A_{FO_3}G_{FO}} = \frac{23.935\times3.855}{2\times.66\times.4\times10^7} = 174.7529\ q$$

119 = .5(θ₂ − .5 sin 2θ₂) = .5(1.74483 − .5[−.34109]) = .957688

120 = .5(θ₂ + .5 sin 2θ₂) = .5(1.74483 + .5[−.34109]) = .787143

121 = 1 − cos θ₂ = 1 − (−.17318) = 1.17318

122 = sin θ₂ = .98489

123 = .5(1 − cos 2θ₂) = .5(1 − [−.94005]) = .97003

124 = 116 θ₂ = 27.1988 q 1.74483 = 47.4573 q

125 = 2 116 θ₂ r_2 = 2 × 27.1988 q 1.74483 × 23.935 = 2,271.78 q

126 = 116 θ₂ r_2^2 = 27.1988 q 1.74483 × 23.935² = 27,187.52 q

127 = 116 119 r_2^2 = 27.1988 q .957688 × 23.935² = 14,922.46 q

128 = 116 120 r_2^2 = 27.1988 q .787143 × 23.935² = 12,265.07 q

129 = 2 116 121 r_2 = 2 × 27.1988 q 1.17318 × 23.935 = 1,527.49 q

130 = 2 116 121 r_2^2 = 2 × 27.1988 q 1.17318 × 23.935² = 36,560.42 q

131 = 2 116 122 r_2 = 2 × 27.1988 q .98489 × 23.935 = 1,282.33 q

132 = 2 116 122 r_2^2 = 2 × 27.1988 q .98489 × 23.935² = 30,692.60 q

133 = 116 123 r_2^2 = 27.1988 q .97003 × 23.935² = 15,114.77 q

134 = 117 119 = 18.1326 q .957688 = 17.364 q

135 = 117 120 = 18.1326 q .787143 = 14.2729 q

136 = 117 123 = 18.1326 q .97003 = 17.5892 q

137 = 118 119 = 174.7529 q .957688 = 167.3588 q

138 = 118 120 = 174.7529 q .787143 = 137.5555 q

139 = 118 123 = 174.7529 q .97003 = 169.5155 q

140 = 126 + 128 − 132 + 135 + 137 = q(27,187.52 + 12,265.07 − 30,692.60 + 14.27 + 167.36) = 8,941.62 q

141 = 127 + 134 + 138 = q(14,922.46 + 17.37 + 137.56) = 15,077.39 q

142 = −125 + 131 = q(−2,271.78 + 1,282.33) = −989.45 q

143 = 130 − 133 − 136 + 139 = q(36,560.42 − 15,114.77 − 17.54 + 169.52) = 21,597.58 q

144 = 124 14^2 = 47.4573 q 44.8^2 = 95,248.70 q

145 = 124 9^2 = 47.4573 q 8.48^2 = 3412.67 q

146 = 2 124 9 14 = 2 × 47.4573 q × 8.48 × 44.8 = 36,058.44 q

147 = 140 115^2 = 8941.62 q $.72697^2$ = 4,725.51 q

148 = 141 114^2 = 15,077.39 q $.68666^2$ = 7,109.02 q

149 = 143 114 115 = 21,597.58 q .68666 × .72697 = 10,781.10 q

150 = 147 + 148 + 149 = 22,615.63 q

151 = 140 142^2 = 8941.62 q $.68666^2$ = 4,215.99 q

152 = 141 115^2 = 15,077.39 q $.72697^2$ = 7,968.17 q

153 = 143 114 115 = 21,597.58 q .68666 × .72697 = 10,781.10 q

154 = 151 + 152 − 153 = 1403.06 q

155 = 2 140 114 115 = 2 × 8941.62 q .68666 × .72697 = 8,926.97 q

156 = 2 141 114 115 = 2 × 15,077.39 q .68666 × .72697 = 15,052.69 q

157 = 143 $(114^2 - 115^2)$ = 21,597.58 q $(.68666^2 - .72697^2)$ = −1,230.69 q

158 = 155 − 156 + 157 = −7,356.41 q

159 = 142 14 115 = −989.45 q × 44.8 × .72697 = −32,224.66 q

160 = 129 14 114 = 1527.49 q 44.8 × .68666 = 46,989.21 q

161 = 159 − 160 = −79,213.87 q

162 = 142 14 114 = −989.45 q 44.8 × .68666 = −30,437.83 q

163 = 129 14 115 = 1527.49 q 44.8 × .72697 = 49,747.69 q

164 = 162 + 163 = 19,309.86 q

165 = 142 9 115 = −989.45 q 8.48 × .72697 = −6089.67 q

166 = 129 9 114 = 1527.49 q 8.48 × .68666 = 8,894.39 q

167 = −165 + 166 = 14,994.06 q

168 = 142 9 114 = −989.45 q 8.48 × .68666 = −5,761.45 q

169 = 129 9 115 = 1527.49 q 8.48 × .72697 = 9,416.53 q

170 = −168 − 169 = −3,655.08 q

171 = 144 + 154 + 164 = q(95,248.70 + 1,403.06 + 19,309.86) = 115,961.62 q

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$172 = 145 + 150 + 167 = q(3412.67 + 22,615.63 + 14,994.06) = 41,022.36\ q$

$173 = 7^2 150 + 6^2 154 - 6\,7\,158 = q(.477426 \times 22,615.63 + .522570 \times 1403.06 + .477426 \times 7356.41) = 15,204.93\ q$ (10,797.29; 733.20; 3674.44)

$174 = -146 + 158^2 + 161 + 170 = q(-36,058.44 - 7356.41 - 79,213.87 - 3,655.08) = -126,283.80\ q$

$175 = 2\,6\,154 - 7(158 + 161) + 6\,164 = q(1.44578 \times 1403.06 + .69096 \times 86,570.28 + .72289 \times 19,309.86) = 75,804.02\ q$ (2028.52; 89,816.60; 13,958.90)

$176 = 6(158 + 170) - 7(150 + 167) = q(-.72289 \times 11,011.49 - .69096 \times 60,225.32) = -49,573.39\ q$ (-7,960.10; -41,613.29)

$177 = a_6 - a_7 = 56.4 - 11.32 = 45.08$

$178 = 14 + a_{29} = 44.8 - 2.1 = 42.7$

$179 = a_5 + a_6 = 17.4 + 19.2 = 36.6$

$180 = 6\,a_{29} + 7\,179 = -.72289 \times 2.1 + .69096 \times 36.6 = 23.77107$ (-1.518069; 25.289176)

$181 = \dfrac{1}{2 E_{FO} I_{FO_3}} = \dfrac{1}{2 \times 10^3 \times 2.19} = .22831\ q$

$182 = \dfrac{a_7}{2 A_{FO_3} E_{FO}} = \dfrac{11.32}{2 \times .86 \times 10^3} = 6.58139\ q$

$183 = \dfrac{a_7 \alpha_{FO_3}}{2 A_{FO_3} G_{FO}} = \dfrac{11.32 \times 1.16}{2 \times .86 \times .4 \times 10^3} = 19.08603\ q$

$184 = 181\,a_7 = .22831\ q \times 11.32 = 2.584470\ q$

$185 = 184\,a_7 = 2.584470\ q \times 11.32 = 29.2562\ q$

$186 = 185\,\frac{a_7}{3} + 183 = 29.2562 \times 3.77333\ q + 19.08603\ q = 129.4793\ q$ (110.3933)

$187 = 184\,178^2 = 2.58447\ q \times 1823.29 = 4712.24\ q$

$188 = 184\,180^2 = 2.58447\ q \times 565.0638 = 1460.39\ q$

$189 = 184\,177^2 = 2.58447\ q \times 2032.2064 = 5252.1765\ q$

$190 = 2\,184\,178\,180 = 5.16894 \times 42.7 \times 23.77107 = 5,246.61\ q$

$191 = 2\,184\,178\,177 = 5.16894 \times 42.7 \times 45.08 = 9,949.79\ q$

$192 = 2\,184\,180\,177 = 5.16894 \times 23.77107 \times 45.08 = 5,539.02\ q$

$193 = 185\,178 = 29.2562\ q \times 42.7 = 1249.24\ q$

$194 = 185\,180 = 29.2562\ q \times 23.77107 = 695.45\ q$

$195 = 185\,177 = 29.2562\ q \times 45.08 = 1318.87\ q$

$196 = 185\,7\,178 = 29.2562\ q \times .69096 \times 42.7 = 863.17\ q$

$197 = 185\,7\,180 = 29.2562\ q \times .69096 \times 23.77107 = 480.53\ q$

$198 = 185\,7\,177 = 20.21446\ q \times 45.08 = 911.29\ q$

$199 = 2\,186\,7 = 2 \times 129.4793\ q \times .69096 = 178.93\ q$

$200 = 186\,7^2 = 129.4793\ q \times .477426 = 61.8168\ q$

$201 = 2\,182\,6 = 2 \times 6.58139 \times .72289 = 9.5152\ q$

$202 = 182\,6^2 = 6.58139 \times .522570 = 3.4392\ q$

$203 = 187 + 182 = 4712.24\ q + 6.58\ q = 4718.82\ q$

$204 = 188 + 197 + 200 + 202 = q(1460.39 + 480.53 + 61.82 + 3.44) = 2,006.18\ q$

$205 = 189 + 195 + 186 = q(5252.1765 + 1318.87 + 128.48) = 6700.53\ q$

$206 = 190 + 196 + 201 = q(5246.61 + 862.17 + 8.52) = 6119.30\ q$

$207 = -191 - 193 = q(-9,949.79 - 1249.24) = -11,199.03\ q$

$208 = -192 - 194 - 198 - 199 = q(-5539.02 - 695.45 - 911.29 - 178.93) = -7324.69\ q$

$209 = 85 + 111 + 171 + 203 = q(4777.07 + 33,782.51 + 115,961.62 + 4718.82) = 159,245.02\ q$

$210 = 173 + 204 = q(15,204.93 + 2006.18) = 17,211.11\ q$

$211 = 59 + 113 + 172 + 205 = q(1835 + 832.38 + 41,022.36 + 6700.53) = 48,173.62\ q$

$212 = 175 + 206 = q(75,804.02 + 6,119.30) = 81,923.32\ q$

$213 = 58 + 112 + 174 + 207 = q(132.44 - 6027.44 - 126,283.80 - 11,199.03) = -143,377.83\ q$

$214 = 176 + 208 = q(-49,573.39 - 7,324.69) = -56,898.08\ q$

$300 = 209 + 46 = q(159,245.02 + 101.95) = 159,346.97\ q$

$301 = 2\ 210 = 34,422.22\ q$

$302 = 2\ 26 = 914.1040\ q$

$303 = 2\ 27 = 23.9786\ q$

$304 = 2\ 300 = 318,693.94\ q$

$305 = 2\ 211 = 96,347.24\ q$

$306 = 2\ 45 = 6739.50\ q$

$UP_1^2 =$	$+ 301$	$=$	$= 34,422.22$
	$+ 2\ 18\ 214$	$= 2 \times .35191 (-56,898.08\ q)$	$= -40,046.01$
	$+ 305\ (18^2 + 24^2)$	$= 96,347.24\ q\ (.123841 + .114965)$	$= 23,007.34$
	$- 2\ 6\ 24\ 213$	$= -2 \times .72289 (-.33905)(-143,377.83)\ q$	$= -70,282.62$
	$+ 6^2\ 304$	$= .522570 \times 318,693.94\ q$	$= 166,539.89$
	$- 2\ 6\ 47\ 52$	$= -1.44578 \times 1034.5178\ q \times .06178$	$= -92.40$
	$+ 306\ (49^2 + 52^2)$	$= 6739.5\ q\ (.0038167 \times 2)$	$= 51.45$
			$113,599.87\ q$

$UP_2 =$	$+ 214\ (19 + 24)$	$= -56,898.08\ q\ (-.33905 \times 2)$	$= 38,582.59$
	$+ 305\ (18\ 19 + 23\ 24)$	$= 96,347.24\ q\ (-.11935 \times 2)$	$= -22,991.34$
	$- 6\ 213\ (23 + 24)$	$= -.72289\ (-143,377.83)(.01286)$	$= 1,332.89$
	$+ 6^2\ 304$	$=$	$= 166,539.89$
	$- 6\ 47\ 212$	$= -.72289 \times 81,923.32$	$= -59,221.55$
	$- 6\ 47\ (53 + 53)$	$= -2\ 6\ 47\ 52$	$= -92.40$
	$+ 306\ (49\ 50 + 52\ 53)$	$= 306\ (49^2 + 52^2)$	$= 51.45$
			$124,201.53\ q$

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Neglecting strain energy of struts and spar.

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205 = 189 + 195 + 186 = Same

206 = 190 + 196 + 201 =

207 = -191 - 193 =

208 = -192 - 194 - 198 - 199 =

209 = 85 + 111 + 171 + 203 =

210 = 173 + 304 =

211 = 59 + 113 + 172 + 205 =

212 = 175 + 206 = Same

213 = 58 + 112 + 174 + 207 = Same

214 = 176 + 208 = Same

300 = 209 + 46 = 209 = 159,245.02 q

301 = 2 210 = Same

302 = 2 26 = 0

303 = 2 27 = 0

304 = 2 300 = 318,490.04 q

305 = 2 211 = Same.

306 = 2 48 = 0

$\infty = A_{ST} = A_{SP} = I_{SP}$

$0 = 26 = 27 = 302 = 303$

$= 32 = 33 = 34$

$= 46 = 47 = 48$

$= 306$

$UP_1^2 =$	+ 301	= ✓	= 34,422.22
	+ 2 18 214	= ✓	= - 40,046.01
	+ 305 (18^2+24^2)	= ✓	= 23,007.34
	- 2 6 24 213	= ✓	= - 70,282.62
	+ 6^2 304	= .522570 × 318,490.04	= 166,433.34
	- 2 6 47 52	=	= ———
	+ 306 (49^2+52^2)	=	= ———
			113,534.27 q

$UP_2 =$	+ 214 (19 + 24)	= ✓	= 38,582.59
	+ 305 (18 19 + 23 24)	= ✓	= -22,991.34
	- 6 213 (23 + 24)	= ✓	= 1332.89
	+ 6^2 304	=	= 166,433.34
	- 6 212	= ✓	= -59,221.55
	- 6 47 (52 + 53)	=	= ———
	+ 306 (49 50 + 52 53)	=	= ———
			124,135.93 q

UP_1Q_1 = + 20 214 = .3968085 (-56,899.05) = -22,577.64
+ 20 305 (18 + 24) = .3968085 × 96,347.24 × .01286 = 491.66
+ 213 (24 - 6 20) = -143,377.83 (-.33905 - .72289 × .3968085) = 89,740.04
- 6 304 = -.72289 × 318,683.84 = -230,380.66
- 47 (6 51 - 52) = -1034.5178 (-.72289 × .08546 - .06178) = 127.83
+ 306 51 (49 + 52) = 6739.5 (-.08546)(.06178 × 2) = - 71.17
-162,669.84 ℔

UP_1Q_2 = + 6 13 47 = .72289 × .0886524 × 1034.5178 = 6.624805
- 13 306 (49 + 52) = -.0886524 × 6739.5 × .12356 = - 7.382378
- .757573 ℔

UP_1R_1 = 20 214 = = -22,577.64
+ 20 305 (18 + 24) = = 491.66
+ 213 (24 - 6 20) = = 89,740.04
- 6 304 = = -230,380.66
- 47 (6 20 - 52) = -1034.5178 (.72289 × .3968085 - .06178) = - 232.84
+ 20 306 (49 + 52) = .3968085 × 6739.5 × .12356 = 330.44
-162,629.00 ℔

UP_1R_2 = -.5 214 = = 28,449.04
-.5 305 (18 - 24) = -48,123.62 (.35191 + .33905) = - 33,286.04
-.5 6 213 = -.361445 (-143,377.83) = 51,823.20
-.5 6 47 = -.361445 × 1034.5178 = -373.92
-.5 306 (49 - 52) = = 0.0
46,612.28 ℔

UP_1R_3 = + 212 = = 81,923.32
+ 213 (18 - 24) = -143,377.83 × .69096 = -99,068.35
+ 6 304 = = 230,380.66
+ 47 (49 - 52) = = 0.0
213,235.63 ℔

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UP_1Q_1 = + 20 214 = ✓ = -22,577.64

+ 20 305 (18 + 24) = ✓ = 491.66

+ 213 (24 - 6 20) = ✓ = 89,740.04

- 6 304 = -.72289 × 318,490.04 = -230,233.27

- 47 (6 51 - 52) = = ———

+ 306 51 (49 + 52) = = ———

= -162,579.21 ₹

UP_1Q_2 = + 6 13 47 = = ———

- 13 306 (49 + 52) = = ———

———

UP_1R_1 = 20 214 = ✓ =

+ 20 305 (18 + 24) = ✓ =

+ 213 (24 - 6 20) = ✓ =

- 6 304 = =

- 47 (6 20 - 52) = = ———

+ 20 306 (49 + 52) = = ———

= -162,579.21 ₹

UP_1R_2 = -.5 214 = ✓ = 28,449.04

-.5 305 (18 - 24) = ✓ = - 33,286.04

-.5 6 213 = ✓ = 51,823.20

-.5 6 47 = = ———

-.5 306 (49 - 52) = = ———

= 46,986.20 ₹

UP_1R_3 = + 212 = ✓ = 81,923.32

+ 213 (18 - 24) = ✓ = - 99,068.35

+ 6 304 = = 230,233.27

+ 47 (49 - 52) = = ———

= 213,088.24 ₹

UP_2^2 =	$+301$	=	= 34,422.22
	$+2\ 23\ 214$	= $2 \times .35191 (-56,894.08)$	= $-40,046.01$
	$-2\ 6\ 212$	= $-1.44578 \times 81,923.32$	= $-118,443.10$
	$+305 (19^2 + 23^2)$	= $96,347.24 (.33905^2 + .35191^2)$	= 23,007.34
	$-2\ 6\ 23\ 213$	= $-1.44578 \times .35191 (-143,377.83)$	= 72,948.35
	$+6^2\ 304$	=	= 166,539.89
	$-2\ 6\ 47\ 53$	=	= $-$ 92.40
	$+306 (50^2 + 53^2)$	=	= 51.45
			138,387.74 ✓

UP_2Q_1 =	$+20\ 214$	=	= $-22,577.64$
	$+212$	=	= 81,923.32
	$+20\ 305 (19 + 23)$	=	= 491.66
	$+213 (23 - 6\ 20)$	= $-143,377.83 (.35191 - .2\&685)$	= $-9,328.15$
	$-6\ 304$	=	= $-230,380.66$
	$-47 (6\ 51 - 53)$	=	= 127.83
	$+51\ 306 (50 + 53)$	=	= $-$ 71.17
			$-179,814.81$ ✓

UP_2Q_2 =	$+6\ 13\ 47$	=	= 6.629805
	$-13\ 306 (50 + 53)$	=	= $-$ 7.382378
			$-$ .752573 ✓

UP_2R_1 =	$+20\ 214$	=	= $-22,577.64$
	$+212$	=	= 81,923.32
	$+20\ 305 (19 + 23)$	=	= 491.66
	$+213 (23 - 6\ 20)$	=	= $-$ 9,328.15
	$-6\ 304$	=	= $-230,380.66$
	$-47 (6\ 20 - 53)$	=	= $-$ 232.84
	$+20\ 306 (50 + 53)$	=	= 380.44
			$-179,773.81$ ✓

UP_2P_2 =	$+.5\ 214$	=	= $-28,449.04$
	$-.5\ 305 (19 - 23)$	=	= 33,286.04
	$-.5\ 6\ 213$	=	= 51,823.20
	$-.5\ 6\ 47$	=	= $-$ 373.92
	$-.5\ 306 (50 - 53)$	=	= 0.0
			56,286.288

AVRO AIRCRAFT LIMITED

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AIRCRAFT

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UP_3^2 = + 301 = = 34,422.22

+ 2 23 214 = = − 40,046.01

− 2 6 212 = = − 118,443.10

+ 305 ($M^2 + 23^2$) = = 23,007.34

− 2 6 23 213 = = 72,948.35

+ 6^2 304 = = 166,433.74

− 2 6 47 53 = =

+ 306 ($50^2 + 53^2$) = =

138,322.14 ℄

UP_2Q_1 = + 20 214 = = − 22,577.64

+ 212 = = 81,923.32

+ 20 305 (19 + 23) = = 491.66

+ 213 (23 − 6 20) = = − 9,328.15

− 6 304 = = − 230,233.27

− 47 (6 51 − 53) = =

+ 51 306 (50 + 53) = =

− 179,724.08 ℄

UP_2Q_2 = + 6 13 47 = =

− 13 306 (50 + 53) = =

UP_2R_1 = + 20 214 = = − 22,577.64

+ 212 = = 81,923.32

+ 20 305 (19 + 23) = = 491.66

+ 213 (23 − 6 20) = = − 9,328.15

− 6 304 = = − 230,233.27

− 47 (6 20 − 53) = =

+ 20 306 (50 + 53) = =

− 179,724.08 ℄

UP_2R_2 = + .5 214 = = − 28,449.04

− .5 305 (19 − 23) = = 33,286.04

− .5 6 213 = = 51,823.20

− .5 6 47 = =

− .5 306 (50 − 53) = =

UP_2R_3 = -212 = = $-81,923.32$
$+213(19-23)$ = $-143,377.83(-.64096)$ = $99,068.35$
$+6\,304$ = = $230,380.66$
$+47(50-53)$ = = 0.0
$247,525.67$ ✓

UQ_1^2 = $2\,20^2\,305$ = $2\times.3968085^2\times96,347.24$ = $30,341.09$
$+2\,20\,213$ = $2\times.3968085\,(-143,377.83)$ = $-113,787.08$
$+304$ = = $318,693.94$
$+2\,47\,51$ = $2\times1034.5178\,(-.08546)$ = -176.82
$+2\,51^2\,306$ = $2\times.08546^2\times6739.50$ = 94.44
$235,164.57$ ✓

UQ_1Q_2 = $-13\,47$ = $-.00886524\times1034.5178$ = -9.171249
$-2\,13\,51\,306$ = $-2\times.00886524(-.08546)(6739.5)$ = $10,212.007$
1.040758 ✓

UQ_1R_1 = $2\,20^2\,305$ = = $30,341.09$
$+2\,20\,213$ = = $-113,787.08$
$+304$ = = $318,693.94$
$+47\,(20+51)$ = $1034.5178(.3968085-.08546)$ = 322.10
$+2\,20\,51\,306$ = $2\times.3968085(-.08546)(6739.5)$ = -457.09
$235,112.76$ ✓

UQ_1R_2 = $.5\,213$ = = $-71,688.92$
$+.5\,47$ = = 517.26
$-71,171.66$ ✓

UQ_1R_3 = -304 = $-318,693.94$ ✓

UQ_2^2 = $2\,13^2\,306$ = $2\times.00886524^2\times6739.5$ = 1.054348 ✓

UQ_2R_1 = $-2\,13\,20\,306$ = $-2\times.00886524\times.3968085\times6739.5$ = -47.416461
$-13\,47$ = $-.00886524\times1034.5178$ = -9.171249
-56.587710 ✓

$UQ_2R_2 = 0 = UQ_2R_3$

UP_2R_1 = -212 = = $-81,923.32$
$+213(19-23)$ = = $99,068.35$
$+6\,304$ = = $230,233.27$
$+47(50-53)$ = = ———
$247,378.30$ g

UQ_1^2 = $2\,20^2\,305$ = = $30,341.09$
$+2\,20\,213$ = = $-113,787.08$
$+304$ = = $318,490.04$
$+2\,47\,51$ = = ———
$+2\,51^2\,306$ = = ———
$235,044.05$ g

UQ_1Q_2 = $-13\,47$ = = ———
$-2\,13\,51\,306$ = = ———
———

UQ_1R_1 = $2\,20^2\,305$ = = $30,341.09$
$+2\,20\,213$ = = $-113,787.08$
$+304$ = = $318,490.04$
$+47(20+51)$ = = ———
$+2\,20\,51\,306$ = = ———
$235,044.05$ g

UQ_1R_2 = $.5\,213$ = = $-71,688.92$
$+.5\,47$ = = ———
$-71,688.92$ g

UQ_1R_3 = -304 = $-318,490.04$

UQ_2^2 = $2\,13^2\,306$ = 0

UQ_2R_1 = $-2\,13\,20\,306$ = = ———
$-13\,47$ = = ———
———

$UQ_2R_2 = 0 = UQ_2R_3$

$UR_1^2 = +302 = -914.10$

$+2\,20^2\,305 = -30,341.09$

$+2\,20\,213 = -113,787.08$

$+304 = -318,693.94$

$+2\,20\,47 = 2 \times .396\,8085 \times 1034.5728 = -821.01$

$+2\,20^2\,306 = 2 \times .396\,8085^2 \times 6734.5 = -2,122.36$

$-239,105.42$ #

$UR_1R_2 = +.5(213 + 47) = -71,171.66$ #

$UR_1R_3 = -304 = +318,693.44$ #

$UR_2^2 = +303 = +23.94$

$+.5(305 + 306) = .5(103,046.74) = +51,543.37$

$51,567.35$ #

$UR_2R_3 = -213 - 47 = 142,343.31$ #

$UR_3^2 = 2\,304 = 637,387.84$ #

$$\left.\begin{aligned} 0 &= u_1 + UR_1^2 R_1 + UR_1R_2 R_2 + UR_1R_3 R_3 \\ 0 &= u_2 + UR_1R_2 R_1 + UR_2^2 R_2 + UR_2R_3 R_3 \\ 0 &= u_3 + UR_1R_3 R_1 + UR_2R_3 R_2 + UR_3^2 R_3 \end{aligned}\right\} \quad u_i = UR_iP_iP_i + UR_iP_2P_2 + UR_iQ_1Q_1 + UR_iQ_2Q_2$$

$0 = u_1 + .023910542 R_1 - .007117166 R_2 - .031869394 R_3$

$0 = u_2 - .007117166 R_1 + .005156735 R_2 + .014234331 R_3$

$0 = u_3 - .031869394 R_1 + .014234331 R_2 + .063738788 R_3$

$0 = 41.82256\,u_1 + R_1 - .297691 R_2 - 1.332856 R_3$

$0 = 140.50536\,u_2 - R_1 + .724549 R_2 + 2.000000 R_3$

$0 = -31.37810\,u_3 + R_1 - .446646 R_2 - 2.000000 R_3$

$.277903 R_2 = -140.50536\,u_2 + 31.3781\,u_3$

$R_2 = -505.5414\,u_2 + 112.9103\,u_3$

$UR_1^2 = +302 =$ —

$+2\,20^2\,306 =$ $= 30,341.09$

$+2\,38\,213 =$ $= -113,787.08$

$+304 =$ $= 318,490.04$

$+2\,20\,47 =$ —

$+2\,20^2\,306 =$ —

$235,044.05\ q$

$UR_1R_2 = +.5(213+47) = .5\,213 = -71,688.92\ q$

$UR_1R_3 = -304 = -318,490.04\ q$

$UR_2^2 = +303 =$ —

$+.5(305+306) = .5\,305 = 48,173.62$

$48,173.62\ q$

$UR_2R_3 = -213-47 = -213 = 143,327.83\ q$

$UR_3^2 = 2\,304 = 636,980.08\ q$

$$0 = U_1 + UR_1^2 R_1 + UR_1R_2 R_2 + UR_1R_3 R_3$$
$$0 = U_2 + UR_1R_2 R_1 + UR_2^2 R_2 + UR_2R_3 R_3$$
$$0 = U_3 + UR_1R_3 R_1 + UR_2R_3 R_2 + UR_3^2 R_3$$

$$U_i = UR_iP_iP_i + UR_iP_2P_2 + UR_iQ_1Q_1 + UR_iQ_2Q_2.$$

$$0 = U_1 + .023504405\,R_1 - .007168892\,R_2 - .031849004\,R_3$$
$$0 = U_2 - .007168892\,R_1 + .004817362\,R_2 + .014333783\,R_3$$
$$0 = U_3 - .031849004\,R_1 + .014333783\,R_2 + .063698008\,R_3$$

$$0 = 42.54521\,U_1 + R_1 - .3050020\,R_2 - 1.3550226\,R_3$$
$$0 = 139.4915\,U_2 - R_1 + .6719811\,R_2 + 2.0000000\,R_3$$
$$0 = -31.39815\,U_3 + R_1 - .4501799\,R_2 - 2.0000000\,R_3$$

$$.221812\,R_2 = -139.4915\,U_2 + 31.39815\,U_3$$
$$R_2 = -628.9032\,U_2 + 141.55487\,U_3$$

Same as 742.5-A except that struts have zero stiffners.
$\infty = 26 = 27 = 302 = 303$

$UR_1^2 = +302 = \infty = \infty$
$+2\,20^2\,305 =$
$+2\,20\,213 =$
$+304 =$
$+2\,20\,47 =$
$+2\,20^2\,306 =$

$UR_1R_2 = +.5(213 + 47) = -71,688.92$ #

$UR_1R_3 = -304 = -318,490.04$

$UR_2^2 = +303 = \infty = \infty$
$+.5(305 + 306) =$

$UR_2R_3 = -213 - 47 = 143,327.83$ #

$UR_3^2 = 2\,304 = 636,980.08$ #

$$\left.\begin{aligned} 0 &= u_1 + UR_1^2 R_1 + UR_1R_2 R_2 + UR_1R_3 R_3 \\ 0 &= u_2 + UR_1R_2 R_1 + UR_2^2 R_2 + UR_2R_3 R_3 \\ 0 &= u_3 + UR_1R_3 R_1 + UR_2R_3 R_2 + UR_3^2 R_3 \end{aligned}\right\} \quad u_i = UR_iP_iP_i + UR_iP_2P_2 + UR_iQ_1Q_1 + UR_iQ_2Q_2$$

$0 = u_1 + \infty R_1 - .00716892 R_2 - .03184900 4 R_3 \quad \therefore R_1 = 0$
$0 = u_2 - .00716892 R_1 + \infty R_2 + .01433783 R_3 \quad \therefore R_2 = 0$
$0 = u_3 - .03184900 4 R_1 + .01433783 R_2 + .06369800 8 R_3$

$\therefore R_3 = -15.69907\, u_3$
$= -15.69907(.02130882 4 P_1 + .02473830 P_2 - .03184900 4 Q_1)$
$= -.3345287 P_1 - .3883609 P_2 + .5000000 Q_1$

Next page is 742.5-F15B

$$-41.82256\,u_1 - 140.50536\,u_2 - .426891\,R_2 = .6671404\,R_3$$

$$\begin{aligned} R_3 &= -62.68933\,u_1 - 210.60850\,u_2 - .639882\,R_2 \\ &= -62.68933\,u_1 - 210.60850\,u_2 + 323.51884\,u_2 - 72.24927\,u_3 \\ &= -62.68933\,u_1 + 112.91034\,u_2 - 72.24927\,u_3 \end{aligned}$$

$$\begin{aligned} R_1 &= 31.37810\,u_3 + .446646\,R_2 + 2\,R_3 \\ &= +31.37810\,u_3 \\ &\quad -225.82038\,u_2 + 50.43093\,u_3 \\ &\quad -125.37866\,u_1 + 225.82068\,u_2 - 144.49854\,u_3 \\ &= -125.37866\,u_1 + .00030\,u_2 - 62.68950\,u_3 \end{aligned}$$

$$\begin{aligned} u_1 &= -.016262900\,P_1 - .017977387\,P_2 + .023511296\,Q_1 - .000005658771\,Q_2 \\ u_2 &= .004661228\,P_1 + .005628628\,P_2 - .007117166\,Q_1 \quad \text{——} \\ u_3 &= .021323563\,P_1 + .024752569\,P_2 - .031869394\,Q_1 \quad \text{——} \end{aligned}$$

$$\begin{aligned} R_1 &= 2.039021\,P_1 + 2.253981\,P_2 - 2.947815\,Q_1 + .000709489\,Q_2 && ++-+ \\ &+ .000001\,P_1 + .000001\,P_2 - .000002\,Q_1 \quad \text{——} && ++-0 \\ &- 1.336764\,P_1 - 1.551726\,P_2 + 1.997876\,Q_1 \quad \text{——} && --+0 \\ &= .702258\,P_1 + .702256\,P_2 - .949939\,Q_1 + .000709489\,Q_2 \end{aligned}$$

$$\begin{aligned} R_2 &= -2.356677\,P_1 - 2.845786\,P_2 + 3.598378\,Q_1 && --+0 \\ &+ 2.407650\,P_1 + 2.794820\,P_2 - 3.598383\,Q_1 && ++-0 \\ &= .050973\,P_1 - .050966\,P_2 \end{aligned}$$

$$\begin{aligned} R_3 &= 1.019510\,P_1 + 1.126990\,P_2 - 1.473907\,Q_1 + .000354745\,Q_2 && ++-+ \\ &+ .526301\,P_1 + .635530\,P_2 - .803602\,Q_1 \quad \text{——} && ++-0 \\ &- 1.540612\,P_1 - 1.788355\,P_2 + 2.302540\,Q_1 \quad \text{——} && --+0 \\ &= .005199\,P_1 - .025835\,P_2 + .025031\,Q_1 + .000354745\,Q_2 \end{aligned}$$

$$-42.54521\,u_1 - 139.4915\,u_2 - .3669791\,R_2 = .6449774\,R_3$$

$$\begin{aligned} R_3 &= -65.96384\,u_1 - 216.27334\,u_2 - .5689794\,R_2 \\ &= -65.96384\,u_1 - 216.27334\,u_2 + 357.83297\,u_2 - 80.54465\,u_3 \\ &= \underline{-65.96384\,u_1 + 141.55963\,u_2 - 80.54465\,u_3} \end{aligned}$$

$$\begin{aligned} R_1 &= 31.39815\,u_3 + .4501799\,R_2 + 2R_3 \\ &= \qquad\qquad\qquad\qquad + 31.39815\,u_3 \\ &\qquad\qquad - 283.11958\,u_2 + 63.727408\,u_3 \\ &\quad -131.92768\,u_1 + 283.11926\,u_2 - 161.089300\,u_3 \\ &= \underline{-131.92768\,u_1 - .00032\,u_2 - 65.96374\,u_3} \end{aligned}$$

$$\begin{aligned} u_1 &= -.01625792\,P_1 - .01797408\,P_2 + .02350440\,Q_1 \\ u_2 &= .00469862\,P_1 + .00566602\,P_2 - .00716889\,Q_1 \\ u_3 &= .02130882\,P_1 + .02473783\,P_2 - .03184900\,Q_1 \end{aligned}$$

$$\begin{aligned} R_1 &= +2.144870\,P_1 + 2.371058\,P_2 - 3.100882\,Q_1 \\ &\quad - .000001\,P_1 - .000001\,P_2 + .000002\,Q_1 \\ &\quad - 1.405610\,P_1 - 1.631800\,P_2 + 2.100879\,Q_1 \\ &= \underline{.739259\,P_1 + .739257\,P_2 - 1.000001\,Q_1} \end{aligned}$$

$$\begin{aligned} R_2 &= -2.954977\,P_1 - 3.563378\,P_2 + 4.508539\,Q_1 \\ &\quad + 3.016474\,P_1 + 3.501884\,P_2 - 4.508541\,Q_1 \\ &= \underline{.061497\,P_1 - .061494\,P_2} \end{aligned}$$

$$\begin{aligned} R_3 &= +1.072435\,P_1 + 1.185528\,P_2 - 1.550441\,Q_1 \\ &\quad + .665135\,P_1 + .802080\,P_2 - 1.014826\,Q_1 \\ &\quad - 1.716312\,P_1 - 1.992600\,P_2 + 2.565267\,Q_1 \\ &= \underline{.021258\,P_1 - .004892\,P_2} \end{aligned}$$

FORM 1108A

AVRO AIRCRAFT LIMITED

TECHNICAL DEPARTMENT (Aircraft)

REPORT NO. 7-0558-8

SHEET NO. 74215 - F15

AIRCRAFT

PREPARED BY M. DATE

CHECKED BY DATE

$$\delta_1 = UP_1^2 P_1 + UP_1P_2 P_2 + UP_1Q_1 Q_1 + UP_1Q_2 Q_2 + UP_1R_1 R_1 + UP_1R_2 R_2 + UP_1R_3 R_3 .$$
$$\delta_2 = UP_1P_2 P_1 + UP_2^2 P_2 + UP_2Q_1 Q_1 + UP_2Q_2 Q_2 + UP_2R_1 R_1 + UP_2R_2 R_2 + UP_2R_3 R_3 .$$
$$\delta_3 = UP_1Q_1 P_1 + UP_2Q_1 P_2 + UQ_1^2 Q_1 + UQ_1Q_2 Q_2 + UQ_1R_1 R_1 + UQ_1R_2 R_2 + UQ_1R_3 R_3 .$$
$$\delta_4 = UP_1Q_2 P_1 + UP_2Q_2 P_2 + UQ_1Q_2 Q_1 + UQ_2^2 Q_2 + UQ_2R_1 R_1 + UQ_2R_2 R_2 + UQ_2R_3 R_3 .$$

$$\delta_1 = .011359987 P_1 + .012420153 P_2 - .016266994 Q_1 - .000000007525 73 Q_2$$
$$-.011420760 P_1 - .011420720 P_2 + .015448876 Q_1 - .000011538 3 \quad Q_2 \qquad --+-$$
$$+.000237597 P_1 - .000237564 P_2 \qquad +-00$$
$$+.000110861 P_1 - .000556894 P_2 + .000533752 Q_1 + .000007564 46 Q_2 \qquad +-++$$
$$= .00028769 P_1 + .00021098 P_2 - .00028445 Q_1 - .00000 40491 Q_2$$

$$\delta_2 = .012420153 P_1 + .013838774 P_2 - .017481481 Q_1 - .000000007525 73 Q_2$$
$$-.012624773 P_1 - .012624773 P_2 + .017077345 Q_1 - .000012754 7 \quad Q_2 \qquad --+-$$
$$+.000288690 8 P_1 - .000288683 P_2 \qquad +-00$$
$$+.000128689 P_1 - .000639483 P_2 + .000 61958 Q_1 + .000000 6784 0822 Q_2 \qquad +-++$$
$$= .00021098 P_1 + .00028769 P_2 - .0002845 Q_1 - .00000 40491 Q_2$$

$$\delta_3 = -.016266994 P_1 - .017981481 P_2 + .023516957 Q_1 + .000000 1040758 Q_2$$
$$+.016510996 P_1 + .016510949 P_2 - .022334297 Q_1 + .000001 66810 \quad Q_2 \qquad ++-+$$
$$-.000362783 P_1 + .000362733 P_2 \qquad -+00$$
$$-.000165689 P_1 + .000823346 P_2 - .000797723 Q_1 - .000001 13055 \quad Q_2 \qquad -+--$$
$$= -.0002845 P_1 - .0002845 P_2 + .0003849 Q_1 + .00000 5480 Q_2$$

$$\delta_4 = -.000000007526 P_1 - .000000007526 P_2 + .000000 104076 Q_1 + .000000 105935 Q_2$$
$$-.000039739 2 P_1 - .000039739 1 P_2 + .000005375487 - .000000 000401 5 Q_2 \qquad --+-$$
$$= -.00000 40491 P_1 - .00000 40491 P_2 + .00000 54796 Q_1 + .00000 01092 Q_2$$

Check: When $P_1 = P_2 = P$ and $0 = \delta_3 = Q_2$,

$$0 = -.0005690 P + .0003849 Q_1$$
$$\therefore Q_1 = 1.4783 P$$
Then $\delta_1 = P(.00028769 + .00021098 - .0002845 \times 1.4783) = .00007809 P.$

When $-P_1 = P_2 = -P$, and $0 = Q_1 = Q_2$,

$$\delta_1 = P(.00028769 - .00021098) = .00007671 P.$$ Ok.

AVRO AIRCRAFT LIMITED

TECHNICAL DEPARTMENT (Aircraft)

AIRCRAFT

REPORT NO. 7-0558-8

SHEET NO. 742.5 + F15A

PREPARED BY — DATE

M.

CHECKED BY — DATE

$$\delta_1 = UP_1^2 P_1 + UP_1P_2 P_2 + UP_1Q_1 Q_1 + UP_1Q_2 Q_2 + UP_1R_1 R_1 + UP_1R_2 R_2 + UP_1R_3 R_3 .$$
$$\delta_2 = UP_1P_2 P_1 + UP_2^2 P_2 + UP_2Q_1 Q_1 + UP_2Q_2 Q_2 + UP_2R_1 R_1 + UP_2R_2 R_2 + UP_2R_3 R_3 .$$
$$\delta_3 = UP_1Q_1 P_1 + UP_2Q_1 P_2 + UQ_1^2 Q_1 + UQ_1Q_2 Q_2 + UQ_1R_1 R_1 + UQ_1R_2 R_2 + UQ_1R_3 R_3 .$$
$$\delta_4 = UP_1Q_2 P_1 + UP_2Q_2 P_2 + UQ_1Q_2 Q_1 + UQ_2^2 Q_2 + UQ_2R_1 R_1 + UQ_2R_2 R_2 + UQ_2R_3 R_3 .$$

$$\delta_1 = .011353427 P_1 + .012413593 P_2 - .016257921 Q_1$$
$$- .012018814 P_1 - .012018782 P_2 + .016257921 Q_1$$
$$+ .000288510 P_1 - .000288369 P_2$$
$$+ .000452830 P_1 - .000104228 P_2$$
$$= .000076547 P_1 + .000001631 P_2$$

$$\delta_2 = .012413593 P_1 + .013832214 P_2 - .017972408 Q_1$$
$$- .013286264 P_1 - .013286228 P_2 + .017972408 Q_1$$
$$+ .000348443 P_1 - .000348426 P_2$$
$$+ .000052587 P_1 - .000012017 P_2$$
$$= .000001649 P_1 + .000076543 P_2$$

$$\delta_3 = -.016257921 P_1 - .017972408 P_2 + .023504405 Q_1$$
$$+ .017375843 P_1 + .017375796 P_2 - .023504405 Q_1$$
$$- .000440865 P_1 + .000440844 P_2$$
$$- .000677046 + .000155605 P_2$$
$$= \text{Zero.}$$

$$\delta_4 = \text{Zero.}$$

Check: When $P_1 = P_2 = P$, $\delta_1 = .000007818\ P$.

When $-P_1 = P_2 = -P$, $\delta_1 = .000007491\ P$.

$$\delta_1 = \underline{UP_1^2}\,P_1 + \underline{UP_1P_2}\,P_2 + \underline{UP_1Q_1}\,Q_1 + \underline{UP_1Q_2}\,Q_2 + \underline{UP_1R_1}\,R_1 + \underline{UP_1P_2}\,R_2 + \underline{UP_1R_3}\,R_3 .$$
$$\delta_2 = \underline{UP_1P_2}\,P_1 + \underline{UP_2^2}\,P_2 + \underline{UP_2Q_1}\,Q_1 + \underline{UP_2Q_2}\,Q_2 + \underline{UP_2R_1}\,R_1 + \underline{UP_2R_2}\,R_2 + \underline{UP_2R_3}\,R_3 .$$
$$\delta_3 = \underline{UP_1Q_1}\,P_1 + \underline{UP_2Q_1}\,P_2 + \underline{UQ_1^2}\,Q_1 + \underline{UQ_1Q_2}\,Q_2 + \underline{UQ_1R_1}\,R_1 + \underline{UQ_1R_2}\,R_2 + \underline{UQ_1R_3}\,R_3 .$$
$$\delta_4 = \underline{UP_1Q_2}\,P_1 + \underline{UP_2Q_2}\,P_2 + \underline{UQ_1Q_2}\,Q_1 + \underline{UQ_2^2}\,Q_2 + \underline{UQ_2R_1}\,R_1 + \underline{UQ_2R_2}\,R_2 + \underline{UQ_2R_3}\,R_3 .$$

$$\delta_1 = +.011353427\,P_1 + .012413593\,P_2 - .016257921\,Q_1$$
$$- .007128413\,P_1 - .008275514\,P_2 + .010654412\,Q_1$$
$$= \underline{.004225013\,P_1 + .004138079\,P_2 - .005603509\,Q_1}$$

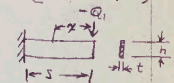
$$\delta_2 = .012413593\,P_1 + .013832214\,P_2 - .017972408\,Q_1$$
$$- .008275514\,P_1 - .009607206\,P_2 + .012368915\,Q_1$$
$$= \underline{.004138079\,P_1 + .004225013\,P_2 - .005603493\,Q_1}$$

$$\delta_3 = -.016257921\,P_1 - .017972408\,P_2 + .023504405\,Q_1$$
$$+ .010654406\,P_1 + .012368908\,P_2 - .015924502\,Q_1$$
$$= \underline{-.005603515\,P_1 - .005603500\,P_2 + .007579903\,Q_1}$$

Checks: 1. When $P_1 = P_2 = P$, and $0 = Q_1$, $\delta_1 = .00836309\,P$ (119.6 #/in.)
$-P_1 = P_2 = -P$, $0 = Q_1$, $\delta_1 = .00006931\,P$ (11,503. #/in.)

2. When $P_1 = P_2 = P$, and $\delta_3 = 0$, $Q_1 = 1.47852\,P$
then, $\delta_1 = (.00836309 - .00828490)P = .00007819\,P$ (12,789 #/in.)

3. If $-Q_1$ is the force at the end of a cantelever beam,



$$\alpha = \frac{A}{It}\int y\,dA = \frac{th}{\frac{t}{12}h^3t} \times \frac{th}{2}\,\frac{h}{4} = 1.5 \quad ; \quad s = 55.5"$$

$$M = Q_1 x$$

$$U = \int_0^s \left\{ \frac{Q_1^2 x^2}{2EI} + \frac{\alpha Q_1^2}{2AG} \right\} dx = Q_1^2 \left(\frac{s^3}{6EI} + \frac{\alpha s}{2AG} \right)$$

Suppose $h = 2(Q_1 + Q_2) = 61.4$ and $t = .031"$

$$\frac{\partial U}{\partial Q} = -\delta_3 = Q_1 \left(\frac{s^3}{3EI} + \frac{\alpha s}{AG} \right) = Q_1 \left(\underbrace{\frac{12 \times 55.5^3 \times 32}{3 \times 10^3 \times 61^3}}_{.000009640\,5} + \underbrace{\frac{1.5 \times 55.5}{\frac{61}{32} \times .4 \times 10^3}}_{.000010918\,0} \right) = .00002056\,Q_1$$

Then $-.0112070\,P = -.007579903\,Q_1 - .00002056\,Q_1 = -.007600463\,Q_1$

$$Q_1 = 1.474515\,P$$

Then, $\delta_1 = (.00836309\,P - .00826246\,P) = .00010063\,P$ (9,937. #/in.)

Referring to the equations on page 742.5-F15, considering the force Φ_1 due to the skin underneath the former as described on page 742.5-F158, and letting $\Phi_2 = 0$, $0 = \delta_3 - \delta_3$

$$-.0002845 P - .0002845 P + .0003849 \Phi_1 + .0002056 \Phi_1 = 0$$

$$\Phi_1 = 1.4032 P$$

$$\delta_1 = P(.00028769 + .00021098 - .00039921) = .00009946 P$$

(10,054 #/in.)

Partial Summary of Results: Station 742.5

Conditions: $P_1 = P_2 = P$. ; $\Phi_2 = 0$.

I_{FO}	I_{SP}	A_{ST}	t	Spec.	P/δ_1
.440	9.42	.3136	0	$0 = \Phi_1 = \Phi_2$	2,005. #/in.
"	"	"	$\frac{1}{32}$"	$\delta_2 = -.0002056 \Phi_1$	10,054.
"	"	"	∞	$\delta_3 = 0$; 0	12,806.
"	∞	∞	0	$\Phi_1 = 0 = \Phi_2$	12,790.
"	∞	0	0	$\Phi_1 = 0 = \Phi_2$	119.
"	∞	0	∞	$\delta_3 = 0$.	12,789.
"	∞	0	$\frac{1}{32}$"	$\delta_3 = -.0002056 \Phi_1$	9937.

SNAP-LOCK BINDER, 1½ inch capacity

NO. 4872-W

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