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SUBJECT SOME SIMPLE CONSIDERATIONS OF WING CAMBER AT HIGH SPEEDS

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ISSUED TO Internal

SUMMARY

This memorandum attempts to deduce the effects of adding arbitrary camber and twist to a wing of given plan form, at least as far as these effects can be determined without involving complex assumptions and lifting surface calculations. The analysis is too simple to provide a method of design of cambered wings, but it does explain qualitatively their behaviour. It also suggests methods of assessing test results, and of comparing the results of cambered wings designed for different flight conditions.

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1.0 INTRODUCTION

There is considerable interest at the present time in the use of camber and twist to reduce the drag coefficient of a high speed wing at non-zero lift coefficients. Various types of camber and twist have been proposed from time to time, and these are discussed for example in References 1, 2 and 3.

Unfortunately a clear physical picture of the effects of surface warping is difficult to obtain by reading such references, because they are concerned mainly with the design of wings to carry specified types of lift distribution, and in order to calculate their drag it is necessary to become involved in complex lifting surface theory.

The present analysis was carried out with only one aim in view, namely, to determine how many of the effects of camber and twist can be deduced by making only a few assumptions which are so simple that a clear physical picture can be retained throughout.

For brevity in what follows, the term "camber and twist" will be reduced to "camber", but a completely arbitrary definition will be assumed.

2.0 ASSUMPTIONS

The following analysis will be concerned with the effects of adding arbitrary camber to a wing of specified plan form. The following basic assumptions will be made:

(a) The wing can be replaced by its median surface, i.e., the surface which splits the wing thickness distribution in half.

(b) The slope of this surface at any point is always small enough that its cosine is unity and its sine is equal to the slope.

(c) The pressure difference acting across the median surface acts at right angles to the surface at every point.

(d) At any angle of attack the pressure difference across the surface is the sum of two parts: a pressure difference due to camber and twist at zero lift, and a pressure difference proportional to angle of attack measured from the wing zero lift angle.

(e) Even when the pressure distribution theoretically approaches infinity at the wing leading edge, no leading edge suction or thrust force is developed which must be subtracted from the drag.

All of the above assumptions, except the last, are in accordance with ordinary linearized wing theory (which is, incidentally the basis used for the design of various wings in References 1, 2 and 3).

No attempt will be made to justify assumption (e) for the present. It will be shown later that the results of the analysis which depend on this assumption are in good agreement with some experimental measurements on cambered delta wings at supersonic speeds, although some other evidence is conflicting.

Assumption (e) is the central one of the present analysis. Leading edge thrust ordinarily arises in linearized thin wing theory whenever the chordwise pressure distributions reach infinity along the wing leading edge, and this situation occurs in general when the wing leading edge is swept to an angle greater than the flow Mach angle. The beneficial effects of camber are sometimes explained by stating that whereas the theoretical thrust force is not realized by a thin flat wing, and "effective" leading edge thrust may be developed by suitable camber. It seemed more logical, however, to derive the effects of camber without reference to this force, which can not exist in reality (at least in exactly the same way as it arises in thin wing theory) because there can not be an infinite pressure at any point on a real wing.

3.0 ANALYSIS

A set of co-ordinates denoted by x, y and z will first be set up to define the wing geometry. Let x be in the direction of the relative wind, i.e., x increases positively in the chordwise direction from leading edge to trailing edge. Let y be the spanwise co-ordinate and z be the vertical co-ordinate measured positively upward in the direction of the lift force.

The median surface of the wing is defined by the functional relation

$$z = f(x,y,\alpha)$$

over the wing plan form, where α is the wing angle of attack.

Let C_p = coefficient of pressure difference across wing at an angle of attack α, at a point (x,y).

$= C_{po} + C_{p\alpha}$

where C_{po} = coefficient of pressure difference across wing at point (x,y) when wing is at zero lift.

$C_{p\alpha} = K_p \times \alpha$ = coefficient of pressure difference across a flat (uncambered) wing of same plan form at point (x,y) when wing is at an angle of attack α.

Let S = wing area.

a = wing lift curve slope = $dC_L/d\alpha$.

The wing lift coefficient, C_L, is given by

$$C_L = \frac{\iint C_p \, dx \, dy}{S}$$

$$= \frac{\iint C_{po} \, dx \, dy}{S} + \frac{\iint C_{p\alpha} \, dx \, dy}{S}$$

$$= \frac{\iint C_{p\alpha} \, dx \, dy}{S} \tag{1}$$

since the first term is, by definition, zero.

Thus,
$$a = \frac{C_L}{\alpha} = \frac{\iint K_p \, dx \, dy}{S} \tag{2}$$

The pressure drag coefficient at zero lift is

$$\Delta C_{D_o} = \frac{-\iint C_{po} \left(\frac{\partial z}{\partial x}\right)_o dx \, dy}{S} \tag{3}$$

where $\left(\frac{\partial z}{\partial x}\right)_o$ denotes the chordwise slope of the wing surface when the wing is at zero lift. This drag coefficient is written in incremental form (with a Δ notation) merely to indicate that this is only the camber contribution to zero lift drag. The skin friction and thickness drag are not dealt with in the present analysis.

When the wing is at an angle of attack, α, the slope at any point on the surface is

$$\frac{\partial z}{\partial x} = \left(\frac{\partial z}{\partial x}\right)_o - \alpha \qquad (4)$$

and the drag coefficient is

$$\Delta C_D = \frac{-\iint C_p \frac{\partial z}{\partial x}\, dx\, dy}{S}$$

$$= \frac{+\iint (C_{po}+\alpha K_p)\left[\alpha - \left(\frac{\partial z}{\partial x}\right)_o\right] dx\, dy}{S}$$

$$= \frac{-\iint C_{po}\left(\frac{\partial z}{\partial x}\right)_o dx\, dy}{S} + \alpha^2 \frac{\iint K_p\, dx\, dy}{S} + \alpha \frac{\iint C_{po}\, dx\, dy}{S}$$

$$- \alpha \frac{\iint K_p \left(\frac{\partial z}{\partial x}\right)_o dx\, dy}{S} \qquad (5)$$

The third term in the above expression is zero by definition. C_{po} is the pressure distribution at zero lift, and hence its integral is zero.

From equations (2) and (3), equation (5) becomes

$$\Delta C_D = \Delta C_{D_o} + \frac{C_L^2}{a} - \alpha \frac{\iint K_p \left(\frac{\partial z}{\partial x}\right)_o dx\, dy}{S} \qquad (6)$$

The last term in the above can be rewritten (from 2):

$$\alpha \frac{\iint K_p\left(\frac{\partial z}{\partial x}\right)_o dx\, dy}{S} = C_L \frac{\iint K_p\left(\frac{\partial z}{\partial x}\right)_o dx\, dy}{\iint K_p\, dx\, dy}$$

The ratio of the two double integrals has an interesting physical interpretation. It is, in effect, a weighted average of the wing surface slope when the wing is at zero lift. The weighting is proportional to the angle of attack pressure distribution, that is, to the pressure distribution of a lifting, uncambered wing of the same plan form. This physical interpretation will be of use later. In the meantime, let the ratio be abbreviated:

$$\frac{\iint K_p \left(\frac{\partial z}{\partial x}\right)_o dx\, dy}{\iint K_p\, dx\, dy} = \overline{\left(\frac{\partial z}{\partial x}\right)_o} \qquad (7)$$

Then,

$$\Delta C_D = \Delta C_{D_o} + \frac{C_L^2}{a} - C_L \overline{\left(\frac{\partial z}{\partial x}\right)_o} \qquad (8)$$

This expression (equation (8)) can be rearranged as follows:

$$\Delta C_D = \Delta C_{D_o} - \frac{a}{4}\overline{\left(\frac{\partial z}{\partial x}\right)_o}^2 + \frac{1}{a}\left[C_L - \frac{a}{2}\overline{\left(\frac{\partial z}{\partial x}\right)_o}\right]^2 \qquad (9)$$

Equation (9) shows that when $C_L = \frac{a}{2}\overline{\left(\frac{\partial z}{\partial x}\right)_o}$ then the drag is a minimum and equal to

$$\Delta C_{D_{min}} = \Delta C_{D_o} - \frac{a}{4}\overline{\left(\frac{\partial z}{\partial x}\right)_o}^2 \qquad (10)$$

Hence

$$\Delta C_D = \Delta C_{D_{min}} + \frac{1}{a}\left(C_L - C_{L_m}\right)^2 \qquad (11)$$

where C_{L_m} = lift coefficient for minimum drag

$$= \frac{a}{z}\overline{\left(\frac{\partial z}{\partial x}\right)_o} \qquad (12)$$

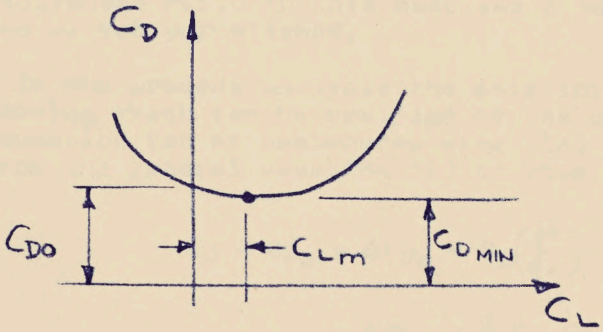
At this point a number of conclusions can be drawn from the results of the analysis.

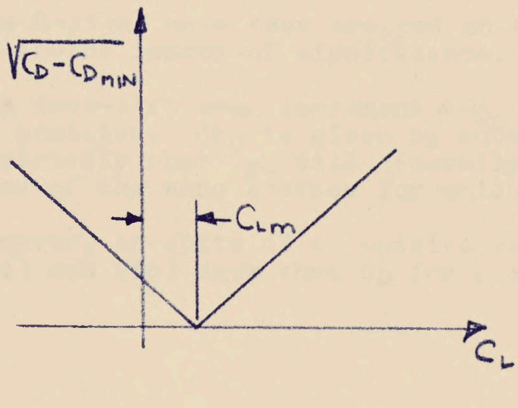
Equation (9) (or (11)) shows that a plot of ΔC_D versus C_L is a parabola. For a flat (uncambered) wing $\overline{\left(\frac{\partial z}{\partial x}\right)_o}$ is zero and in this case the drag equation becomes simply,

$$\Delta C_{D_\mu} = \frac{C_L^2}{a} \qquad (13)$$

since ΔC_{D_o} is zero from equation (3). This result is well-known for flat wings which develop no leading edge thrust force. Moreover equation (9) shows that for a cambered wing the curvature of the parabolic drag polar is the same as that of an uncambered wing of the same plan form. The polar is shifted relative to that of a flat wing by an amount C_{L_m} in the lift coefficient direction and by an amount ΔC_{D_m} in the drag direction.

Equation (11) also suggests that a convenient way to plot drag polars for cambered wings would be in the form of C_D versus $(C_L - C_{L_m})^2$ or alternatively in the form of $\sqrt{C_D - C_{D_{min}}}$ versus C_L, since in either case straight line plots should result. The latter form is undoubtedly preferable since $C_{D_{min}}$ is much easier to read from a graph of C_D versus C_L than is C_{L_m}. Furthermore a plot of $\sqrt{C_D - C_{D_{min}}}$ versus C_L should be in the form of a V-shaped pair of straight lines which intersect on the C_L axis at the value $C_L = C_{L_m}$. The following sketches should make these points clearer.





The implications of equation (11) are that the slope of the straight line segments on the latter graph should be the same for a cambered wing as for an uncambered wing of the same plan form, and equal to $\sqrt{1/a}$. This point has been checked with the data given for delta wings in Reference 3 and has been found to be true to within about 5 percent in all cases at supersonic speeds. This limited comparison, however, should not be taken as implying a general rule. There is some data available from other sources which conflicts with this result. Reference 4 contains measurements at transonic speed on a 3 percent thick delta wing with and without camber. The wing aspect ratio was 3.0. These results show a low value of drag due to lift, which implies that considerable leading edge suction was being realized up to a Mach number of 1.38 by the uncambered wing and not by the cambered wing, at least at some speeds. The close approach to full leading-edge suction at the higher Mach numbers was pointed out in Reference 5 as being contrary to usual assumptions in linearized theory, but no explanation was given. A further set of conflicting data has been obtained by the NAE High Speed Aerodynamics Laboratory in tests of a half model delta wing-body combination of aspect ratio 2. With one percent negative circular arc camber this wing has a higher drag due to lift than the uncambered wing. The wing thickness ratio in this case was 3 percent. These results are as yet unpublished.

In the present analysis the main interest is in the drag saving which can be realized by the use of camber. The drag equation for an uncambered wing (13) may be subtracted from the general equation (8) to give

$$C_D - C_{D_\mu} = \Delta C_{D_o} - C_L \left(\overline{\frac{\partial z}{\partial x}}\right)_o \qquad (14)$$

$$= \Delta C_{D_o} - \frac{2}{a} C_L C_{L_m} \qquad (15)$$

The Δ-signs have been dropped on the left hand side because they are no longer of significance.

The zero-lift drag increment ΔC_{D_o} due to camber will generally be positive. C_{D_o} is given by equation (3), and it is known intuitively that C_{p_o} will generally be negative over those portions of the wing surface for which $\frac{\partial z}{\partial x}$ is positive.

However, in spite of a positive value of ΔC_{D_o}, equations (14) and (15) show that C_D for a cambered wing should be lower than that of a similar uncambered wing for sufficiently large lift coefficients, provided only that the quantity $\left(\overline{\frac{\partial z}{\partial x}}\right)_0$ is positive.

The quantity $\left(\overline{\frac{\partial z}{\partial x}}\right)_0$ is defined in equation (7), where it was interpreted as being an average value of the wing surface slope at zero wing lift, weighted in proportion to the pressure distribution of an equivalent flat wing at an angle of attack.

In the case of a flat wing having a quasi-subsonic pressure distribution with high pressure differences near the leading edge, the weighting will be heaviest near the leading edge and the weighted average slope $\left(\overline{\frac{\partial z}{\partial x}}\right)_0$ of the equivalent cambered wing will be determined mainly by the surface slope in the leading edge region. For a wing with positive camber (concave downward) this implies that $\left(\overline{\frac{\partial z}{\partial x}}\right)_0$ is positive, and from equation (14) the implication is that positive camber is required for the reduction of drag at positive lift coefficients. This result is a well known empirical fact.

To carry this sort of physical reasoning further, it follows that a change of Mach number which moves the aerodynamic centre of a flat wing rearward, or which causes a reduction in the leading edge suction peaks of a flat wing, will reduce the benefits available from a given amount of camber, since in either case $\left(\overline{\frac{\partial z}{\partial x}}\right)_0$ will be reduced. It appears to be borne out experimentally, (see for example Reference 5) that an increase of Mach number through the transonic range (rearward motion of aerodynamic centre) or through the region where the Mach lines are approaching and crossing the wing leading edge (reduction in pressure peaks) does indeed reduce greatly the benefits to be derived from camber.

Equations (14) and (15) imply that if the drag difference between a cambered wing and a similar flat wing are plotted against lift coefficient, the result will be a straight line which eventually crosses over the C_L axis. Negative values of the drag difference indicate lower drag for the cambered wing than for its flat equivalent.

4.0 CHOICE OF OPTIMUM CAMBER

Consider now a family of wings all having the same plan form and the same basic camber shape, but different amounts of camber. The question arises as to the amount of camber which will give the greatest drag reduction at a specified lift coefficient and Mach number. Within the limitations of the present analysis some conclusions can be reached on this point.

Dimensional considerations applied to equation (3) will show that ΔC_{D_o}, the drag increment due to camber, is proportional to the square of the amount of camber. Similarly, equation (12) shows that C_{L_m}, the lift coefficient for minimum drag is directly proportional to amount of camber. Thus the quantity $\Delta C_{D_o}/C_{L_m}^{\ 2}$ may be expected to be a constant for any family of cambered wings under consideration.

Equation (15) can be rewritten so that this parameter appears more explicitly:

$$C_D - C_{D_\mu} = \left(\frac{\Delta C_{D_o}}{C_{L_m}^{\ 2}}\right) C_{L_m}^{\ 2} - \frac{2}{a}\, C_L\, C_{L_m} \qquad (16)$$

The value of C_{L_m} for which this expression is a minimum for a given C_L, may be obtained by setting

$$\frac{\partial (C_D - C_{D_\mu})}{\partial C_{L_m}} = 0,$$

and the result is

$$\left(C_{L_m}\right)_{design} = \frac{1}{a}\left(\frac{C_{L_m}^{\ 2}}{\Delta C_{D_o}}\right) C_{L_{design}} \qquad (17)$$

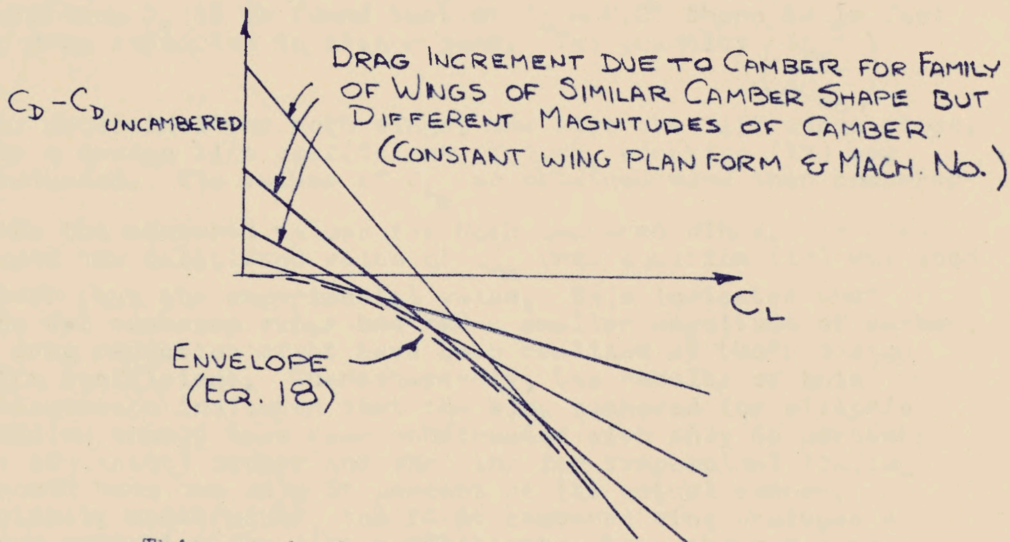
If this is substituted into (16) the result is

$$C_D - C_{D_\mu} = -\frac{C_L^{\ 2}}{a^2}\left(\frac{C_{L_m}^{\ 2}}{\Delta C_{D_o}}\right) \qquad (18)$$

Equation (18) shows that the maximum possible drag reduction due to camber is proportional to the square of the design lift coefficient, and hence is small for low design lift coefficients.

The possible drag reduction also is proportional to the parameter $\dfrac{C_{L_m}^{\ 2}}{\Delta C_{D_o}}$ which appears to be independent of magnitude of camber at a given Mach number. This parameter may therefore be used as a basis of comparison between wings of different basic camber shape, even if the wings being compared were designed for different lift coefficients. A further implication is that the search for good camber shapes might be narrowed to the search for the basic shape having the highest value of $\frac{C_{L_m}^2}{\Delta C_{D_0}}$ at the Mach number of interest.

The results given by the above considerations may be summarized graphically as follows:



This graph shows that camber chosen to reduce drag as much as possible at high lift coefficients may result in large drag penalties at low lift coefficients. There will therefore be a tendency in any practical case to choose camber on the low side in order to effect an acceptable compromise at all flight conditions.

There appears to be some indication that cambered wings designed using linearized theory may have too large a magnitude of camber to achieve the maximum possible drag saving at their design lift coefficients. The method of design is usually based on the assumption that a cambered wing will realize an "effective" leading edge suction if, at its design lift coefficient, the pressure distribution goes to zero along the leading edge. This requires a rather large amount of camber, however, such that the zero lift drag penalty may be too large to permit much drag reduction.

To test this guess, equation (17) was applied as a criterion to the experimental results from two cambered wings in Reference 3. These two wings were aspect ratio 2 deltas with 3 percent maximum thickness, one of which was cambered and twisted for elliptic loading and the other for trapezoidal loading. In each case the theoretical design lift coefficient was 0.25 at M = 1.53. When C_D is plotted versus C_L for these wings at M = 1.53, and compared with the results of the uncambered delta which are also given in Reference 3, it is found that at $C_L = 0.25$ there is in fact no drag reduction in either case. The quantity $\left(\frac{{C_{L_m}}^2}{\Delta C_{D_o}}\right)$ was determined for both wings, and also the lift curve slope. For a design lift coefficient of 0.25, equation (17) was evaluated. The values of C_{L_m} so obtained were then compared with the measured values for both cambered wings. In both cases the calculated value of C_{L_m} from equation (17) was much lower than the experimental value. This indicates that if the two cambered wings had had a smaller magnitude of camber, a drag reduction might have been realized at their design lift coefficient. Quantitatively, the results of this calculation indicated that the wing cambered for elliptic loading should have been constructed with only 63 percent of its actual camber and the wing for trapezoidal loading should have had only 57 percent of its actual camber. As actually constructed, the first cambered wing produced a drag reduction for lift coefficients above about 0.3 and the second wing had the same drag as the uncambered wing for lift coefficients above about 0.4. At lower lift coefficients, and in particular at 0.25 there was a drag penalty.

It may therefore be concluded that a type of camber designed using linearized theory should not be rejected on the basis of wind tunnel tests merely because these show no drag gain due to damber at the design operating conditions. It may be that a considerable reduction in the magnitude of camber would result in a reduction in drag at the design lift coefficient.

5.0 CONCLUSIONS

This analysis has been based on the ordinary assumptions of linearized wing theory, with the exception that no leading edge suction or thrust force has been assumed either for cambered or flat wings. The following conclusions have been reached without involving complex lifting surface calculations.

1. The curve of drag coefficient versus lift coefficient for a wing with arbitrary camber and twist is a parabola of the same shape as for a flat wing of the same plan form, but is displaced in both the lift and drag direction from that of the flat wing.

2. In general the addition of camber (and/or twist) will increase the zero lift drag coefficient of a wing by an amount ΔC_{D_o}, which is proportional to the square of the amount of camber (and/or twist).

3. The lift coefficient, C_{L_m}, for minimum drag is directly proportional to the amount of camber. Hence the quantity $C_{L_m}{}^2/\Delta C_{D_o}$ is a constant for a particular type of camber applied to a particular plan form, and is independent of magnitude of camber. It will, of course, depend on Mach number.

4. Basically different types of camber tested on the same wing plan form can be compared by comparing the value of $C_{L_m}{}^2/\Delta C_{D_o}$, which should be as high as possible. The maximum possible saving in drag due to camber is directly proportional to this parameter.

5. Experimental drag polars for cambered wings are conveniently plotted in the form $\sqrt{C_D - C_{D_{min}}}$ versus C_L. The slope of the straight lines which result should be the same for cambered and uncambered wings. Comparison of this result with available measurements shows agreement in some cases at supersonic speeds but not in others.

6. The difference in drag coefficient between a cambered and an uncambered wing of the same plan form should be linear with lift coefficient.

7. At sufficiently high positive lift coefficients, the drag of a positively cambered (concave downward) wing should be less than that of an uncambered wing.

Since this result has nothing to do with the theoretical leading edge suction force, it should hold even at Mach numbers such that the Mach lines are swept behind the wing leading edge.

8. The possible reduction in drag by means of camber should decrease as Mach number is increased through the transonic range and through the range where the Mach lines are swept over the leading edge. Hence, the higher the Mach number, the less is likely to be the reduction of drag due to camber.

9. The maximum possible drag reduction due to camber is proportional to the square of the lift coefficient.

10. For any given lift coefficient there is an optimum amount of camber which will effect the maximum possible drag saving. Too large a magnitude of camber may be particularly bad since it will produce a drag penalty at lift coefficients somewhat below the design value.

11. There are indications that wings designed using linearized theory to carry specified types of loading which will ensure zero loading along the leading edge, have too large a magnitude of camber for their design lift coefficient. This is because the definition of design lift coefficient used in such methods is not necessarily that at which the maximum possible drag reduction will be achieved. A set of cambered delta wings designed and tested by the NACA (Reference 3) produced no drag reduction below that of a flat wing at their theoretical design conditions. The results of the present analysis, when applied to these experimental data, indicated that the wings in question should have had some 40 percent less camber (and twist), and that under these conditions, a drag saving would have occurred at the design lift coefficient.

6.0 REFERENCES

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